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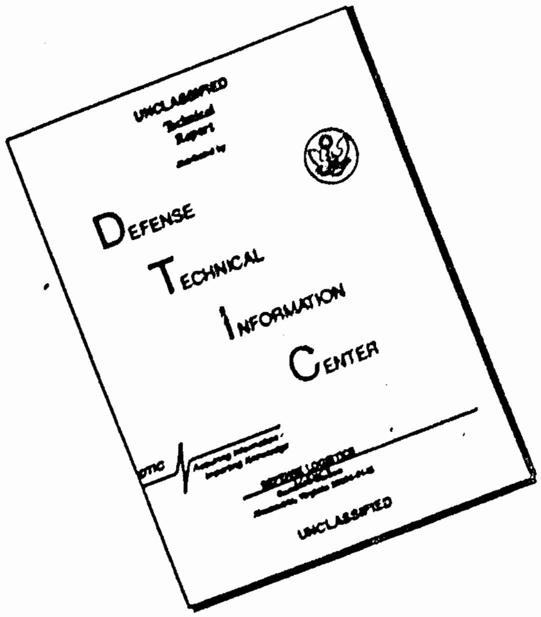


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SSD-TDR-64-159

REPORT NO.
TDR-269(4110-04)-1



# TRACE

## Aerospace Orbit Determination Program

NOVEMBER 1964



*Prepared by*
R. J. MERCER, et al.
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*Electronics Division*

*Prepared for* COMMANDER SPACE SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
*Los Angeles, California*



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CONTRACT NO. AF 04(695)-269

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AEROSPACE ORBIT DETERMINATION PROGRAM

Prepared by

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El Segundo Technical Operations
AEROSPACE CORPORATION
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TRACE

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This technical documentary report is approved for publication and dissemination. The conclusions and findings herein do not necessarily represent an official Air Force position.

H. L. Norwood, Jr.
Colonel, USAF
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## FOREWORD

TRACE was conceived in August 1961, to meet the Aerospace Corporation requirements in the fields of satellite tracking and system design. Although under continuing development, the program has been used extensively in post-flight orbit determination and tracking system analysis.

The present report is the first "complete" description of the program; the previous partial descriptions issued in March and December 1962 are now obsolete and should be discarded. The information herein should be sufficient for most users of the program.

TRACE was designed by M. Bennett, R. J. Mercer, D. Morrison, L. Sachnoff, and C. C. Tonies; the principal contributors to the program include the originators and D. A. Adams, C. Christensen, D. Groves, K. Hubbard, S. McDonald, J. Ostlie, and A. Skulich.

## ABSTRACT

TRACE is a multiple-purpose satellite orbit-determination program for the IBM 7090 computer. Its applications include: (1) prediction - the generation of a satellite trajectory and associated ground trace and station sighting data; (2) orbit determination - estimation of trajectory parameters, station locations, and observational biases, so as to best fit a set of observations; and (3) error analysis - estimation of the potential accuracy attainable by a tracking system, given the station locations, the data types, rates and quality, the uncertainties in the model parameters, and the specifications of the nominal orbit. The report contains the objectives of the program, some theoretical foundations, the equations and methods employed, the structure of the program, and complete instructions for its use.

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## FIGURES

## 1.2 TRAJECTORY GENERATION

Basic to all applications of TRACE is the space vehicle trajectory. A trajectory is defined by a set of initial conditions together with the differential equations of motion which reflect the earth's gravitational and atmospheric forces and those of other bodies affecting the vehicle's motion. In TRACE, the trajectory is generated in an inertial rectangular coordinate system by a step-by-step numerical integration. (Between the integration points, the trajectory is defined by an interpolation formula.)

### 1.2.1 Trajectory-Related Output

Trajectory-related output may be obtained from any application of TRACE at any reasonable set of time points, as well as optionally at the points of equator crossings, apogee, and perigee. Any or all of three blocks of information may be selected for output at print times. They are:

a. Basic trajectory information including position and velocity components, spherical coordinates, ground trace, and altitude.

b. Conic section elements computed from the components of position and velocity.

c. Partial derivatives of position and velocity components with respect to initial conditions and differential equation parameters. (These quantities are necessary for the other applications of TRACE but have also been found useful in simple trajectory generation where effects of initial condition or parameter errors are sought.)

### 1.2.2 Required Input

a. Epoch - the date and time of injection.

b. Initial conditions of the orbit. Three types are acceptable: (1) inertial rectangular components of position and velocity, (2) spherical coordinates for position, together with the flight path angle, azimuth, and magnitude of the velocity vector, and (3) elements of a conic section. In (2), either the right ascension (inertial) or the longitude (referred to Greenwich) may be specified.

c. The drag parameter $C_D A/W$ and a choice of atmosphere model. (The ARDC 1959 model is used unless otherwise specified.)

1.2.3 Optional Input

a. Print times and output block indicators. (Trajectory-related output is optional for any of the applications of TRACE; in the simple trajectory generator function, this output is presumably the purpose of the run.)

b. If partial derivatives with respect to certain trajectory parameters are required, these parameters must be specified.

c. Many model constants and numerical integration parameters have been assigned standard values. All of them may be changed by optional input.

## 1.3 RADAR DATA GENERATION

Simultaneous with trajectory generation, TRACE can produce listings of satellite rise and set times, radar coordinates, and many related quantities for up to 50 radar stations,* provided, of course, that the location and characteristics of the stations are supplied. During visibility periods (determined by the program), the desired output quantities are computed in chronological order and stored in the memory until capacity is reached, at which point the information is sorted by station and output. The process is repeated as necessary to complete the listings. Optionally the eight quantities, through Q in the following paragraph, may be written on a magnetic tape in chronological order in the format of tracking input data.**

The quantities to be output (in the station listings or on the data tape) are selected by input, and include range, azimuth, elevation, range rate, doppler data ($\dot{P}$, $\dot{Q}$, P and Q), azimuth rate, elevation rate, range acceleration, mutual visibility (up to eight stations only), latitude, longitude, surface range, altitude, doppler rate, look angle, and standard deviations of the six observational quantities R, A, E, $\dot{R}$, $\dot{A}$, $\dot{E}$.

### 1.3.1 Required Input

a. Station location data.

b. Control information for each station, such as the minimum and maximum elevation angles, maximum range of visibility, the interval (during visibility periods) at which computations are to be made, and the start and stop times for visibility testing and output.

c. The list of output quantities desired from each station.

---

*"Radar station" should be interpreted here generally as a point on the surface of the earth associated with satellite observations. It could be a camera location, or the location of a point observed from the satellite.

**Optionally, random noise may be added to the same eight observational quantities.

1.3.2 Optional Input

a. Control flags to indicate that only rise and set times are to be generated, and that the generated observational quantities are to be listed chronologically on a magnetic tape in the format of input data.

b. The mean and standard deviation of normally distributed random noise to be added to the generated observational quantities.

c. The computed value of elevation is altered to account for atmospheric refraction. A numerical coefficient may be changed, or set to zero if no correction is desired, by optional input.

d. Uncertainties in the initial conditions of a trajectory will be reflected in uncertainties in generated observational quantities. A variance-covariance matrix of initial conditions must be input if the computations of the standard deviations of observational quantities are selected.

## 1.4 TRACKING

Approximately, TRACE can determine the trajectory that best fits a set of observations.

### 1.4.1 The Tracking Problem

More precisely, the trajectory of a space vehicle depends upon the initial conditions of the motion and the differential equation parameters which appear in the equations of motion. From the trajectory, one may compute at the observation times the expected values of the recorded observations. This computation further depends upon the locations of the radar stations and biases in the observations. Thus, the computed "observations" are functions of parameters of four types: initial condition, differential equation, station, and observation parameters. The tracking problem is to solve for the set of parameters that minimizes the differences between the computed and measured observations.

Therefore, TRACE is able to solve for such quantities as the ballistic coefficient (a differential equation parameter) of the vehicle, the location of an observing station, and the presence of observational or time biases in the data reported by a station, in addition to the usual initial condition parameters. (In practice, one solves only for a selected set of parameters rather than all possible parameters.)

### 1.4.2 The Tracking Problem Solution

The solution is an iterative process. Initial estimates of each of the parameters must be provided. Based on these estimates, the "computed observations" and their partial derivatives with respect to the parameters are formed, the normal matrix is accumulated, and measured and computed observations are differenced, forming the "residuals." The residuals are weighted by a combined scale and quality factor, checked against an editing criterion, and the sum of the squares of the weighted residuals is accumulated. When all of the observations have been so treated, a correction to the set of parameters is computed and applied, and the process is repeated. The root mean square of the weighted residuals provides the measure of convergence of the process.

The solution parameters, which are derived from observations containing random errors, must be regarded as estimates of the true parameters. Under certain conditions (that the observational model is correct, that the observational errors are independently distributed with mean zero and variance $\sigma^2$, and that the weighting factor used is $\sigma^{-1}$) the inverse of the normal matrix is the variance-covariance matrix of the parameters. Thus the solution process provides an estimate of the uncertainties in the derived parameters.

Two types of conditions may be imposed upon the solution of the minimization problem: bounds upon the magnitude of the computed corrections may be given, and linear constraints among the corrections may be specified. The former is used to assure covergence, and the latter may represent physical requirements, such as the fact that the difference in the latitudes of two stations is accurately known.

The output includes the rms of the residuals (optionally the residuals may also be reported by station) for the current iteration, the current and corrected values of the parameters, the rms residual that is predicted for the next iteration, and the standard deviations of, and the correlations among, the parameters (obtained from the variance-covariance matrix).

Further optional output includes the individual residuals, partial derivatives of observations with respect to parameters, and trajectory information at observation times.

1.4.3 Required Input

a. The list of initial condition, differential equation, station location, and observational parameters for which the program is to solve, an initial estimate of each, and a bound (if necessary) upon the magnitude of correction that is to be permitted

b. Locations of all observation stations

c. The observational data. Many types of data are acceptable, but eight of these (range, azimuth, elevation, range rate, and the four doppler quantities, $\dot{P}$, $\dot{Q}$, P, Q) are regarded as basic; the other data types are first converted to the above set.

d. Weighting factors for the basic data types for each observing station

1.4.4 Optional Input

a. The maximum number of iterations in the differential correction process may be specified.

b. A refraction correction is applied to elevation observations. A coefficient in this correction may be modified, or set to zero, by optional input.

c. The names of up to nine stations for which residuals are to be reported may be input.

d. If linear constraints among the parameters exist, a constraint matrix must be input.

e. The level of residuals above which data points are to be discarded may be input.

## 1.5 STATISTICAL ANALYSIS

As already mentioned, under certain assumptions the inverse of the normal matrix contains information as to the uncertainties with which parameters are determined by a tracking system. This aspect of orbit determination is exploited in TRACE to provide its system analysis capability. (Note that the normal matrix involves only partial derivatives, not residuals, and thus the performance of a system can be analyzed without recourse to actual observations.)

### 1.5.1 Assumptions

The assumptions are that observational errors are independently distributed with mean zero and variance $\sigma^2$, and that $\sigma^{-1}$ is used as the weighting factor in forming the normal matrix. Under these conditions, the inverse of the normal matrix is a variance-covariance matrix of the parameters being estimated.

Insofar as these parameters are differential equation or radar parameters, their variances and covariances satisfactorily describe their uncertainties. However, the uncertainties in the motion of a vehicle are not adequately described by the variances and covariances of the initial conditions and differential equation parameters of the trajectory. Rather, trajectory uncertainties are better reported in terms of orbit plane coordinates, or conic section elements, or such related quantities as period, apogee, and perigee distance. Cartesian and spherical coordinate variance-covariance matrices are also available.

A further sophistication arises from the assumption that the values of some of the parameters used in the calculations, but <u>not</u> being estimated by the differential correction process, are also uncertain, thereby inducing uncertainties in the differentially corrected parameters and in the trajectory. (This is a very common situation; most tracking programs do not solve for basic constants and station locations, but their current values must be somewhat uncertain.) Such parameters are referred to as "Q-parameters" in distiction to "P-parameters," which are those being estimated by differential correction. TRACE will simultaneously report P-parameter and trajectory uncertainties with and without Q-parameter effects. The matrix of derivatives of the P-parameters with respect to the Q-parameters can also be output.

### 1.5.2 Mechanics of Application

The mechanics of this application of TRACE are as follows: as the trajectory is generated, the program determines (and outputs) periods of visibility from each station. At the prescribed interval, while the vehicle is visible, the partial derivatives of the observations with respect to the P- and Q-parameters are computed, weighted, and accumulated (in double precision) into the normal matrix. At the specified output points, the desired covariance matrices are computed and output, along with such trajectory-related quantities as may have been selected.

### 1.5.3 Required Input

a. The list of up to 30 parameters, of which up to 15 may be trajectory parameters

b. The set of output times and the list of output variance-covariance matrices desired. Optionally, only the standard deviations (square root of the diagonal elements) can be printed.

c. Station location information

d. Control information, for each station, such as the minimum and maximum elevation angles, maximum range of visibility, the interval (during visibility periods) at which computations are to be made, and the start and stop times for visibility testing and output

e. The list of data types reported by each station. Eleven types are possible: range, azimuth, elevation, range rate, the four doppler quantities ($\dot{P}$, $\dot{Q}$, P, Q), argument of the latitude, the orthogonal angle measured from the equator to the position of the vehicle in the plane containing the radius vector and the vector normal to the orbit plane, and geocentric distance. (The last may be used to simulate altitude observations, as height and geocentric distance have a nearly constant difference.)

f. Standard deviations for each type of data

### 1.5.4 Optional Input

a. If Q-parameter effects are desired, each parameter must be designated as being of type P or Q.

b. An input covariance matrix is required for the set of Q-parameters.

SECTION 2

THEORY

## 2.1 INTRODUCTION

Section 1 contains general descriptions of the various applications of TRACE to orbit determination problems. In Section 2, more precise mathematical statements of these problems are provided and the capabilities of TRACE are discussed. The emphasis in this section, however, is upon theoretical aspects and functional relations; the particular equations and methods used in the program are set forth in Section 3. Again, the applications are treated in the order of increasing scope, but not with uniform thoroughness. Topics that are possibly less familiar have been emphasized, whereas more familiar problems, such as the numerical solution of differential equations, have been largely ignored.

## 2.2 THE TRAJECTORY AND ITS PARTIAL DERIVATIVES

The trajectory of a space vehicle is defined by the (differential) equation of motion

$$\ddot{X} = -\frac{\mu X}{r^3} + F \tag{1}$$

together with the initial values $X(t_o) = X_o$ and $\dot{X}(t_o) = \dot{X}_o$. Here. X is a 3-vector of rectangular components (x, y, and z) of position in an inertial coordinate system, a dot represents a time derivative, $r = |X| = (x^2 + y^2 + z^2)^{1/2}$, $\mu$ is the gravitational constant (GM) of the earth, and F (a vector) represents the perturbing accelerations upon the vehicle.

One application of TRACE is merely to solve this differential equation. The solution $X(t)$, $\dot{X}(t)$ is generated numerically at time points $t = t_j$ (j = 0, 1, 2, . . . ) and defined at $t \neq t_j$ by an interpolation formula.

The more sophisticated applications of TRACE require the sensitivity (as expressed by partial derivatives) of the trajectory to its initial conditions and other parameters.

Obviously $\ddot{X}$ is a function of $\mu$ which is an example of a "differential equation parameter." Other such parameters (ballistic coefficients, oblateness coefficients, etc.) may appear in F. Furthermore, the solution depends on the initial conditions $X_o$ and $\dot{X}_o$, which in turn may be computed from "initial condition parameters." If we let vectors of these types of parameters be represented by $\beta$ and $\alpha$ respectively, we may show the functional relations in Eq. (1) as

$$\ddot{X} = -\frac{\mu X}{r^3} + F(X, \dot{X}, \beta, t) \tag{2}$$

or

$$\ddot{X} = \ddot{X}(X, \dot{X}, \beta, t)$$

with

$$X(t_o) = X_o(\alpha) \qquad , \qquad \dot{X}(t_o) = \dot{X}_o(\alpha) \tag{2a}$$

(F and $\ddot{X}$ will be functions of $\dot{X}$ whenever drag forces are present.) The dependence of the solution upon the parameters can be indicated as

$$X(t) = X(\alpha, \beta, t_o, t)$$

and similarly for $\dot{X}(t)$. (In fact, the solution can be given by the integral equations

$$\dot{X}(\alpha, \beta, t_o, t) = \dot{X}_o(\alpha) + \int_{t_o}^{t} \ddot{X}[X(\alpha, \beta, t_o, t''), \ \dot{X}(\alpha, \beta, t_o, t''), \beta, t''] \, dt''$$

and

$$X(\alpha, \beta, t_o, t) = X_o(\alpha) + \int_{t_o}^{t} \dot{X}(\alpha, \beta, t_o, t') \, dt'$$

$$= X_o(\alpha) + (t - t_o)\dot{X}_o(\alpha)$$

$$+ \int_{t_o}^{t} \int_{t_o}^{t'} \ddot{X}[X(\alpha, \beta, t_o, t''), \ \dot{X}(\alpha, \beta, t_o, t''), \ \beta, t''] \, dt'' dt'$$

$$X(t) = X_o(\alpha) + (t - t_o)\dot{X}_o(\alpha)$$

$$+ \int_{t_o}^{t} (t - t'') \ddot{X}[X(\alpha, \beta, t_o, t''), \ \dot{X}(\alpha, \beta, t_o, t''), \ \beta, t''] \, dt'' \tag{3}$$

which are hardly suitable for computations, but which do show the functional relations more explicitly.)

We are now in a position to show how partial derivatives $\frac{\partial X}{\partial \alpha}$, $\frac{\partial X}{\partial \beta}$, and $\frac{\partial X}{\partial t_o}$, which measure the sensitivity (to first order) of solutions to variations in the trajectory parameters $\alpha$, $\beta$, and $t_o$, are obtained. (These partial derivatives are extensively used in other applications of TRACE, but are also often of interest in their own right.)

If we differentiate Eqs. (2) and (2a) with respect to $\alpha$, interchange orders of differentiation, and use the notation $X_\alpha$ for $\frac{\partial X}{\partial \alpha}$, we obtain

$$\frac{\partial \ddot{X}}{\partial \alpha} = \left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right]\frac{\partial X}{\partial \alpha} + \frac{\partial F}{\partial \dot{X}}\frac{\partial \dot{X}}{\partial \alpha}$$

or

$$\ddot{X}_\alpha = \left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right]X_\alpha + \frac{\partial F}{\partial \dot{X}}\dot{X}_\alpha \tag{4}$$

with initial conditions $X_\alpha(t_o) = \frac{\partial X_o}{\partial \alpha}$ and $\dot{X}_\alpha(t_o) = \frac{\partial \dot{X}_o}{\partial \alpha}$. (See Paragraph 2.2.1). Equation (4) is called a "variational equation." It is obviously a second-order linear vector differential equation whose solution is the vector of partial derivatives $X_\alpha = \frac{\partial X}{\partial \alpha}$ of the components of position with respect to the initial condition parameter $\alpha$. In the course of solving Eq. (4), $\dot{X}_\alpha = \frac{\partial \dot{X}}{\partial \alpha}$ will also be obtained. Such an equation can be derived for each initial condition parameter.

One can also obtain Eq. (4) by differentiating the integral Eq. (3) with respect to $\alpha$,

$$X_\alpha = X_\alpha(t_o) + (t - t_o)\dot{X}_\alpha(t_o) + \int_{t_o}^{t}(t - t'')\left(\frac{\partial \ddot{X}}{\partial X}\frac{\partial X}{\partial \alpha} + \frac{\partial \ddot{X}}{\partial \dot{X}}\frac{\partial \dot{X}}{\partial \alpha}\right)dt'' \quad , \tag{5}$$

and noting that Eq. (5) corresponds to Eq. (4) in exactly the same way that Eq. (3) corresponds to Eq. (2). Reference 1, which initiated the authors' use of variational equations, follows the integral formulation, but is more concerned with interplanetary applications.

The variational equations for initial time $t_o$ are of the same form, but with different initial conditions

$$\ddot{X}_{t_o} = \left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right] X_{t_o} + \frac{\partial F}{\partial \dot{X}} \dot{X}_{t_o} \quad ,$$

$$X_{t_o}(t_o) = -\dot{X}_o(a) \quad , \quad \dot{X}_{t_o}(t_o) = -\ddot{X}_o(a) \quad .$$

These are derived by differentiating the integral equation (Eq. 3), which best shows the dependence upon $t_o$, with respect to $t_o$.

The variational equations for a differential equation parameter $\beta$ are

$$\ddot{X}_\beta = \left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right] X_\beta + \frac{\partial F}{\partial \dot{X}} \dot{X}_\beta + \frac{\partial F}{\partial \beta} \quad ,$$

$$X_\beta(t_o) = \dot{X}_\beta(t_o) = 0 \quad . \tag{6}$$

As a source of partial derivatives, variational equations give results that are more accurate than analytic derivatives (which assume two-body motion), and are more rapidly generated than difference quotient approximations. The greater speed derives from the fact that the terms $\left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right]$ and $\frac{\partial F}{\partial \dot{X}}$ of Eq. (4) are identical in all the variational equations; only the non-homogeneous term $\frac{\partial F}{\partial \beta}$ of Eq. (6) varies with the particular parameter.

A further advantage of the variational equations is that they permit the use of the difference quotient technique as a checking device. The two methods must produce partial derivative estimates that are in substantial agreement; the lack thereof would indicate the presence of a blunder. While the test is hardly foolproof, it is valuable and should not be overlooked.

### 2.2.1 Derivative with Respect to a Vector

The indication of a derivative with respect to a vector is a very convenient notational device for representing partial derivative matrices and chain rule differentiation. The following conventions are observed throughout:

a. A "vector" is a column vector; a row vector will be described as such or denoted as a transposed vector. Example: $(x, y, z) = X^T$.

b. The derivative of a vector with respect to a scalar is a vector.

c. The derivative of a scalar with respect to a vector is a row vector.

d. The derivative of a vector with respect to a vector is a matrix.

Example: If F is a vector function of a vector variable X, then $\frac{\partial F}{\partial X}$ is the matrix of partial derivatives whose i - j$^{th}$ element is $\frac{\partial F_i}{\partial X_j}$.

Note the neatness of the following example: Suppose X(t) is the vector $[x_1(t), x_2(t), \ldots, x_n(t)]^T$ and y is a scalar function of X; $y = f[x_1(t), x_2(t), \ldots, x_n(t)] = f[X(t)]$. Then

$$\frac{dy}{dt} = \frac{\partial f}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial f}{\partial x_2}\frac{dx_2}{dt} + \ldots + \frac{\partial f}{\partial x_n}\frac{dx_n}{dt} = \frac{\partial f}{\partial X}\frac{dX}{dt} \quad . \tag{7}$$

Here $\frac{\partial f}{\partial X}$ is by convention a row vector, $\frac{dX}{dt}$ a column vector, and the juxtaposition of the two indicates the desired scalar product.

## 2.3 BASIC ORBIT DETERMINATION

The basic orbit determination problem, as outlined in Section 1.4, is that of finding values for a set of parameters from an observational model so as to minimize, in the sense of weighted least squares, the differences between the measured observations and the corresponding quantities computed from the model.

The model, as constituted in TRACE, includes the trajectory of the vehicle (and thus the initial condition and differential equation parameters), the locations of the observing stations, and constant bias errors in their instruments or their clocks. In practice, one determines values for only a selected subset, P, of the parameters of the model.

The weighting factors are necessary to assign the proper relative significance to observations of different types and quality.

The basic orbit determination problem may now be restated: Given a set of n normalized observations (multiplied by an appropriate weighting factor), which are collectively denoted by the n-vector $O_m$ (m for "measured"), and a model from which the corresponding (similarly weighted) quantities $O_c$ can be computed as functions of parameters P, determine values of P so that $||O_m - O_c(P)||^2$ is minimized.

Suppose that an approximate solution $P_o$ is known. (Approximate initial conditions will be available either from design information or preliminary orbit determination methods.) We expand $O_c(P)$ in a Taylor series to first order about $P_o$ and obtain

$$||O_m - O_c(P)||^2 = ||O_m - O_c(P_o) - A \cdot \Delta P||^2 \tag{8}$$

to be minimized, where $A = \frac{\partial O_c}{\partial P}$ is a matrix of normalized partial derivatives evaluated at $P = P_o$. The partial derivatives, with respect to trajectory parameters, are computed from the chain rule formula $\frac{\partial O_c}{\partial P} = \frac{\partial O_c}{\partial X} \frac{\partial X}{\partial P}$ where $\frac{\partial X}{\partial P}$ is the matrix of solutions to the variational equations. The matrix $\frac{\partial O_c}{\partial X}$ and those columns of $\frac{\partial O_c}{\partial P}$ that represent derivatives with respect to station parameters are computed directly from geometrical relations.

The differences, $O_{mc}(P_o) = O_m - O_c(P_o)$, between the normalized observations and the corresponding quantities computed from assumed values $P_o$ are called "residuals"; they will be due to the presence of random observational errors, inadequacies in the model, and incorrect values for the model parameters.

The above statement of the weighted least squares (WLS) problem conceals the weighting factors, which could have been explicitly included in the formulation as the elements of a diagonal matrix $W^{1/2}$. (This notation is chosen in order to simplify subsequent equations in which $(W^{1/2})^T W^{1/2} = W$ appears frequently.) Then the quantity to be minimized would have been $||W^{1/2}(O_{mc} - A \cdot \Delta P)||^2$, with $O_{mc}$ and A representing actual, not weighted, values. Since TRACE is restricted to independent observations for which W is diagonal, we have chosen (for the sake of simplicity, not deception) to include the weighting factors with the elements of $O_{mc}$ and A. In Section 2.5 on statistical aspects, the weighting matrix is discussed and displayed.

It should be noted that the solution of the WLS problem does not produce "true" values for model parameters -- it produces only "best fit" values. Any further conclusions of a statistical nature regarding the WLS solution require additional assumptions regarding the model and the character of the random observational errors. These topics will be discussed later.

Inasmuch as the original nonlinear WLS problem has been replaced by an approximate linear problem (that of finding a correction vector $\Delta P$ so that $||O_{mc}(P_o) - A \cdot \Delta P||^2$ is minimized), we must not expect that $P = P_o + \Delta P$ will be a solution of the original problem; rather, an iterative process is indicated. $||O_{mc}(P_o)||$ measures the degree to which an orbit, computed from the current values $P_o$ of the parameters, fits the observations. $||O^p_{mc}|| = ||O_{mc}(P_o) - A \cdot \Delta P||$ is an approximation (based upon the linearity assumption) to the value of $||O_{mc}||$ that would be obtained by replacing $P_o$ with $P_o + \Delta P$. (The superscript p means "predicted.") In a well-behaved iteration, the observed $||O_{mc}||$ should follow the predicted $||O^p_{mc}||$, and relative agreement of the two is a measure of convergence of the process.

The correction vector $\Delta P$ is found as the solution of the linear system

$$(A^T A)\Delta P = A^T O_{mc} \quad . \tag{9}$$

This may be shown in various ways, of which two proofs follow.

Proof 1

Let

$$f(\Delta P) = ||A \cdot \Delta P - O_{mc}||^2 = (A \cdot \Delta P - O_{mc})^T (A \cdot \Delta P - O_{mc})$$

Differentiate $f(\Delta P)$ with respect to $\Delta P$. The result is

$$\frac{\partial f}{\partial(\Delta P)} = 2\left(A^T A \cdot \Delta P - A^T O_{mc}\right)^T$$

which must be zero if $\Delta P$ minimizes $f(\Delta P)$.

Proof 2 -

Let

$$A^T A \cdot \Delta P = A^T O_{mc}. \quad \text{Then for any } \Delta P' \neq \Delta P$$

$$
\begin{aligned}
f(\Delta P') &= ||A \cdot \Delta P - O_{mc} + A(\Delta P' - \Delta P)||^2 \\
&= ||A \cdot \Delta P - O_{mc}||^2 + 2[A \cdot \Delta P - O_{mc}]^T[A(\Delta P' - \Delta P)] \\
&\quad + ||A(\Delta P' - \Delta P)||^2 \\
&= f(\Delta P) + 2\left(A^T A \cdot \Delta P - A^T O_{mc}\right)^T(\Delta P' - \Delta P) \\
&\quad + ||A(\Delta P' - \Delta P)||^2 \\
&= f(\Delta P) + ||A(\Delta P' - \Delta P)||^2 \\
&> f(\Delta P)
\end{aligned}
$$

from which we see that $\Delta P$ minimizes $f(\Delta P)$.

## 2.4 CONSTRAINED AND BOUNDED LEAST SQUARES SOLUTIONS

Two distinct types of restrictions upon the solution of the weighted least squares problem may be necessary or desirable. Constraints among the parameters may be a part of the physical problem, and bounds upon the magnitude of the corrections may be computationally desirable.

### 2.4.1 Constraints

An example of a physical constraint among parameters would be precise knowledge of the relative locations of two nearby radar stations.* If their actual locations were among the parameters P in a differential correction, it would be important to constrain the corrections $\Delta P$ so that the radar relative locations were preserved. This is accomplished in TRACE by introducing linear constraints in the form

$$\Delta P = B \cdot \Delta P' + C \tag{10}$$

where B is a rectangular matrix and $\Delta P'$ is a reduced set of independent parameters; by solving the WLS problem in terms of $\Delta P'$; and by using Eq. (10) to obtain the constrained corrections $\Delta P$. Solving the WLS problem in terms of $\Delta P'$ requires the minimization of $||A \cdot \Delta P - O_{mc}||^2$ subject to the constraint Eq. (10), or therefore the minimization of $||A \cdot (B \cdot \Delta P' + C) - O_{mc}||^2 = ||(AB) \cdot \Delta P' - (O_{mc} + AC)||^2$. The solution of the linear system

$$(AB)^T (AB) \Delta P' = (AB)^T (O_{mc} + AC) \tag{11}$$

gives the required minimum.

### 2.4.2 Bounds

Under fairly common conditions, such as inadequacies in the observational model or a poor initial approximation $P_o$, the observed $||O_{mc}||$ will fail to

---

*Radar station may refer to any point on the surface of the earth associated with an observation.

follow the predicted $||O^P_{mc}||$ or may even diverge. In the presence of such manifestations of nonlinearity, it may be necessary, in order to assure eventual convergence, to solve the WLS problem at each iteration with a side condition bounding the magnitude of the correction vector $\Delta P$.

If we refer to the reciprocals of the bounds $g_i$ collectively as the diagonal matrix G, the restricted problem becomes that of minimizing $||A \cdot \Delta P - O_{mc}||^2$ subject to the bounding condition $||G \cdot \Delta P||^2 \leq 1$. (If $||G \cdot \Delta P||^2 = \sum_i \left(\frac{\Delta p_i}{g_i}\right)^2 \leq 1$, then for each component $|\Delta p_i| \leq g_i$.)

The constraint has, in a two-parameter example, a simple geometrical description. The constrained problem is to find a minimum, over all $\Delta P$ within the ellipse defined by $g_1$ and $g_2$, of the surface $f(\Delta P) = ||A \cdot \Delta P - O_{mc}||^2$. (See Figure 1.)

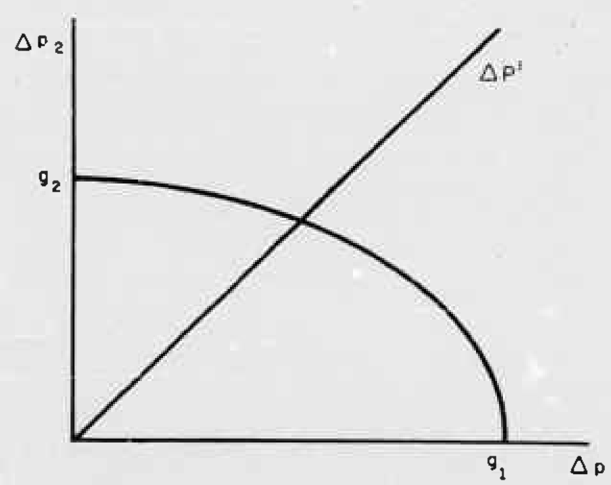



Figure 1. Two-Parameter Constraint Ellipse

(An elliptic rather than circular region is used to account for the range of magnitudes of the various parameters.)

If the unconstrained solution is not within the ellipse $||G \cdot \Delta P||^2 = 1$, then we invent a new function $F(\Delta P) = f(\Delta P) + z||G \cdot \Delta P||^2$ to be minimized. The minimum point $\Delta P'(z)$ is found as the solution of

$$(A^T A + zG^T G)\Delta P = A^T O_{mc} \tag{12}$$

As z increases, the minimization of F will require smaller and smaller values $\Delta P'(z)$. (More precisely, it will be shown that $||G \cdot \Delta P'(z)||$ is a decreasing function of z.) In particular we can find, by a search and interpolation procedure, a value $z'$ of z such that $||G \cdot \Delta P'(z')||^2 = 1$. But for $z'$, the minimization of $F = f + z' ||G \cdot \Delta P'(z')|| = f + z'$ is equivalent to minimizing f, since they differ only by the constant $z'$. Thus we have found the point $\Delta P'(z')$ which minimizes $f(\Delta P)$ along the bounding ellipse.

We will also show that $f(\Delta P'(z))$ is an increasing function of z. Thus, any interior point of the ellipse corresponds to larger values of z and of f, and therefore the constrained minimum point is on the boundary and is the solution $\Delta P'(z')$ of $(A^T A + z' G^T G)\Delta P = A^T O_{mc}$ for which $||G \cdot \Delta P'(z')||^2 = 1$.

The monotonic decreasing character of $||G \cdot \Delta P||$ as a function of z is shown as follows. (Primes have been dropped throughout these proofs.) If we differentiate

$$(A^T A + zG^T G)\Delta P(z) = A^T O_{mc} \tag{13}$$

with respect to z we obtain

$$(A^T A + zG^T G)\frac{d}{dz}(\Delta P) + (G^T G)\Delta P = 0 \tag{14}$$

or

$$\frac{d}{dz}(\Delta P) = -(A^T A + zG^T G)^{-1}(G^T G)\Delta P \qquad (15)$$

This expression is needed in the equation for $\frac{d}{dz}||G \cdot \Delta P||^2$, which will be shown to be negative for all $z \geq 0$. We have

$$\frac{d}{dz}||G \cdot \Delta P||^2 = 2(\Delta P^T)(G^T G)\left(\frac{d}{dz}\Delta P\right)$$

$$= -2\Delta P^T(G^T G)(A^T A + zG^T G)^{-1}(G^T G)\Delta P \qquad (16)$$

Since $(A^T A + zG^T G)^{-1}$ is positive definite for positive $z$,

$$\frac{d}{dz}||G \cdot \Delta P||^2 < 0 \qquad (17)$$

whenever $(G^T G) \cdot \Delta P \neq 0$.

The monotonic increasing character of $f[\Delta P(z)]$ is similarly established by showing that $\frac{df}{dz} > 0$, as follows:

$$\frac{df}{dz} = \frac{\partial f}{\partial(\Delta P)} \frac{d(\Delta P)}{dz}$$

$$= \left[2\left(A^T A \Delta P - A^T O_{mc}\right)\right]^T \left[-(A^T A + zG^T G)^{-1}(G^T G)\Delta P\right]$$

$$= -2\left[(A^T A + zG^T G)\Delta P - A^T O_{mc} - z(G^T G)\Delta P\right]^T \left[(A^T A + zG^T G)^{-1}(G^T G)\Delta P\right]$$

(18)

But since $(A^TA + zG^TG)\Delta P = A^TO_{mc}$,

$$\frac{df}{dz} = +2z\Delta P^T(G^TG)(A^TA + zG^TG)^{-1}(G^TG)\Delta P \qquad (19)$$

and $\frac{df}{dz} > 0$, whenever $(G^TG)\Delta P \neq 0$.

### 2.4.3 Solution of the Linear System

The solution of the linear system $(A^TA + zG^TG)\Delta P = A^TO_{mc}$ (and the inversion of the coefficient matrix $C = A^TA + zG^TG$) is accomplished by a special method akin to that known classically as the square root method. (See Reference 2.) It is a finite (noniterative) method, applicable only to symmetric matrices, and is based on the fact that a symmetric matrix can be decomposed as a product of the form $C = LDL^T$ where $L$ is a lower triangular matrix with (-1) as diagonal elements, and D is a diagonal matrix.

In such a representation det (L) = ±1 and det (D) = det (C). Therefore $L^{-1}$ exists (and also has the form of L) and D has no zero elements if C is nonsingular. Therefore two equivalent forms are

(1) $L^{-1}C(L^T)^{-1} = L^{-1}C(L^{-1})^T = D$

(2) $C^{-1} = (L^{-1})^TD^{-1}L^{-1}$

or

(1′) $SCS^T = D$

(2′) $C^{-1} = S^TD^{-1}S$ where $S = L^{-1}$

Thus we see that the inversion of C and the solution $\Delta P' = (C^{-1})A^TO_{mc}$ require matrices S and D such that (1′) $SCS^T = D$.

A bordering technique is used to find S and D. At the $k^{th}$ stage suppose that the $k^{th}$ order principal minors of S and D have been found. The $(k+1)^{st}$ order minors require the vector W and the scalar b so that

$$\begin{pmatrix} S_k & 0 \\ W^T & -1 \end{pmatrix} \begin{pmatrix} C_k & d \\ d^T & \alpha \end{pmatrix} \begin{pmatrix} S_k^T & W \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} D_k & 0 \\ 0 & b \end{pmatrix} \tag{20}$$

where $C_k$ is the $k^{th}$ and $\begin{pmatrix} C_k & d \\ d^T & \alpha \end{pmatrix}$ the $(k+1)^{st}$ order minors of C. It is easily verified that the required W and b are

$$W = S_k^T D_k^{-1} S_k d$$

and

$$b = \alpha - W^T d \tag{21}$$

## 2.5 THE STATISTICS OF ORBIT DETERMINATION

In the process of orbit determination by the method of weighted least squares (WLS), no assumptions regarding the statistics of the observational errors need be made. In this case, no statistical conclusions can be drawn from the results, and the justifications of the method are simply that it minimizes residuals (in the sense of WLS), and that it works in practice.

On the other hand, if two common assumptions are made, namely (a) that the observational errors $\epsilon_i$ are independent with mean zero and known variance $\sigma_i^2$, and (b) that the multiplicative weighting factor associated with each observation is $\sigma_i^{-1}$, then the inverse normal matrix is a variance-covariance matrix (often abbreviated "covariance matrix") of the parameters being determined. This matrix depends only on the partial derivatives of the observations with respect to the parameters, thus permitting statistical analysis of a tracking network in the absence of actual or simulated observations. The details are covered in the next few sections. The relation of WLS orbit determination, as in TRACE, to other criteria (minimum variance and maximum likelihood) is also discussed.

### 2.5.1 The Variance-Covariance Matrix

We assume that the vector of measured observations $O_m$ is the true value $O_c(P_t)$ plus a random error $\epsilon$. Our linear approximation to $O_c(P)$ is

$$O_c(P) = O_c(P_o) + A \cdot \Delta P \tag{22}$$

and the residual vector is

$$O_{mc} = O_m - O_c(P_o) = A \cdot \Delta P_t + \epsilon \tag{23}$$

where

$O_c(P_o)$ = the vector of computed quantities

$P_o$ = an estimate of the true parameter vector $P_t$

$\Delta P_t = P_t - P_o$

A = the matrix of partial derivatives $\frac{\partial O_c}{\partial P_o}$

$\epsilon$ = the vector of observational errors

with $E(\epsilon) = 0$ and $E(\epsilon\,\epsilon^T) = \Sigma$, a matrix representing the variances and covariances of the observational errors. The WLS problem is the minimization of $f(\Delta P) = ||O_{mc} - A \cdot \Delta P||^2$, wherein each component of $O_{mc}$ and $A \cdot \Delta P$ has been multiplied by a prescribed weighting factor. The solution, as noted before, is $\Delta P' = (A^T A)^{-1} A^T O_{mc}$.

If we call the individual weights $w_i$ and collect them in a diagonal matrix $W^{1/2}$, then the above formulas, with the weighting matrix now explicitly displayed, become

$$f(\Delta P) = ||W^{1/2} O_{mc} - W^{1/2} A \cdot \Delta P||^2 \tag{24}$$

and

$$\Delta P' = (A^T W A)^{-1} A^T W O_{mc} \tag{25}$$

First, we show that $\Delta P'$ is an <u>unbiased</u> estimate of the true value $\Delta P_t$. (By this we mean that although $\Delta P'$ is a random quantity since it depends upon the residuals, and thus upon the observational errors, the <u>expected</u> <u>value</u> of

$\Delta P'$ is the true value $\Delta P_t$.)

$$\Delta P' = (A^T W A)^{-1} A^T W O_{mc}$$

$$= (A^T W A)^{-1} A^T W(A \cdot \Delta P_t + \epsilon)$$

$$= \Delta P_t + (A^T W A)^{-1} A^T W \epsilon$$

$$E(\Delta P') = \Delta P_t + E[(A^T W A)^{-1} A^T W \epsilon] = \Delta P_t \qquad (26)$$

by the linearity of $E(\cdot)$ and on the assumption that $E(\epsilon) = 0$.

The vector $\delta P' = \Delta P' - \Delta P_t$ would be the deviation, due to random errors, of the solution $\Delta P'$ from the true value $\Delta P_t$; it has been shown to have the expected value zero.

What is now the expected value of the square of the deviations or of the product of two components thereof? The answers are summarized in $E(\delta P' \delta P'^T)$, which is by definition the estimated covariance matrix $C(P')$ of the parameters,

$$E(\delta P' \delta P'^T) = E[(A^T W A)^{-1} A^T W(\epsilon \epsilon^T) W A (A^T W A)^{-1}] \quad , \qquad (27)$$

in which we used the symmetry of the matrices $W$ and $A^T WA$, or

$$C(P') = (A^T W A)^{-1} A^T W \Sigma W A (A^T W A)^{-1} \qquad (28)$$

This is the general form of the covariance matrix for a WLS estimate of the parameters. If, however, $\Sigma$ is diagonal, as per the first assumption, making it possible to choose $W = \Sigma^{-1}$, as per the second assumption, to be the diagonal weighting matrix, then the very great simplification

$$C(P') = (A^T W A)^{-1} = (A^T \Sigma^{-1} A)^{-1} \tag{29}$$

is the result. This is the basic covariance matrix calculated in TRACE.

### 2.5.2 Minimum Variance and Maximum Likelihood Estimates

In most instances of orbit determination from observations, the method of weighted least squares (WLS) requires no statistical justification; indeed there is none. Its aim is simply to produce fits and predictions of acceptable quality. In other applications, such as systems analysis and design, statistical conclusions are sought. These are commonly based on minimum variance (MV) or maximum likelihood (ML) estimations. (MV is also called "Markov.") The purpose of this section is to describe in general terms the assumptions governing MV and ML techniques and to relate them to the basically WLS method in TRACE.

The MV, or Markov, estimate of $\Delta P$ is that linear unbiased estimate that minimizes the diagonal terms (the variances) of the variance-covariance matrix of parameters P. (See Reference 3.) The formulas are

$$\Delta P_{MV} = (A^T \Sigma^{-1} A)^{-1} A^T \Sigma^{-1} O_{mc} \tag{30}$$

and

$$C(P_{MV}) = (A^T \Sigma^{-1} A)^{-1} \quad . \tag{31}$$

When $\Sigma^{-1}$ is diagonal and is used as the weighting matrix W, the WLS estimate and covariance matrix in TRACE is also MV or Markov.

Nothing so far has been assumed about the actual form of the distribution of the random errors. If a specific probability density function is assumed, then it is possible to seek the estimate that maximizes the probability or likelihood of the resulting residuals. In the case of a joint normal (or gaussian) distribution of observational errors with covariance matrix $\Sigma$, the maximum likelihood (ML) estimate reduces to MV. (In practice, as Magness and McGuire note in Reference 4, the gaussian assumption is always made. Reference 4 also contains an excellent comparison of LS and MV estimation.)

In summary, for uncorrelated observational errors, the WLS estimate in TRACE is also MV; if the errors are further assumed to be normally distributed, the estimate is also ML.

### 2.5.3 Q-Parameters

Observations are functions of many parameters, including six orbital parameters, differential equation parameters such as drag and spherical harmonic coefficients, radar station locations, and observational biases. In principle, all such parameters can be estimated, given sufficient observations, and the covariance matrix reports the accuracy with which they have been or could be determined. In practice, however, only a selected set of these are estimated. There arises then, both in actual orbit determination and in systems analysis, the question of the effect on (a) the parameters P being estimated, or (b) the trajectory itself, of errors or uncertainties in the remaining parameters Q. The treatment here follows that of Magness and McGuire in Reference 5.

In TRACE, the computation of Q-parameter effects is restricted to the error analysis link FEIGN, in which P-parameter and trajectory covariance matrices, with and without Q-parameter uncertainties, are computed and printed. The transformation from orbital (initial condition) parameters to trajectory coordinates is covered in Section 2.5.4.

Now the true observation vector $O_c(P_t, Q_t)$ is a function of true but unknown values of both P and O and the measured vector is

$$O_m = O_c(P_t, Q_t) + \epsilon \quad . \tag{32}$$

The vector of residuals, in the "measured-minus-computed" sense, is

$$\begin{aligned} O_{mc} &= O_m - O_c(P_o, Q_o) \\ &= O_c(P_t, Q_t) + \epsilon - O_c(P_o, Q_o) \quad . \end{aligned} \tag{33}$$

As in Section 2.5.1, we assume a model for the true observations $O_c(P_t, Q_t)$, which is linear in a correction $\Delta P_t$ to an approximate value $P_o$

$$O_c(P_t, Q_t) = O_c(P_o, Q_t) + A_p \Delta P_t \quad . \tag{34}$$

But now in forming the "computed" quantities we are uncertain as to the true value of the Q parameters and must use an approximate value $Q_o$ in the calculations. Thus $O_c(P_o, Q_o)$ is related to $O_c(P_o, Q_t)$ by

$$O_c(P_o, Q_o) = O_c(P_o, Q_t) - A_q \Delta Q_t \quad . \tag{35}$$

Collecting these results we have the following representation of the residual vector $O_{mc}$

$$O_{mc} = A_p \Delta P_t + A_q \Delta Q_t + \epsilon \quad . \tag{36}$$

(This equation could have been presented more briefly; the longer presentation is used to make clear the sources of the terms in the residual vector.)

Summarizing the notation above, $O_m$ and $O_c$ are the measured and computed observations,

$P_o$ and $Q_o$ = estimates of the true parameter values $P_t$ and $Q_t$

$$\Delta P_t = P_t - P_o \text{ and } \Delta Q_t = Q_t - Q_o$$

$A_p$ and $A_q$ = the matrices $\frac{\partial O_c}{\partial P_o}$ and $\frac{\partial O_c}{\partial Q_o}$, respectively, and

$\epsilon$ = the vector of observational errors with covariance matrix $E(\epsilon\,\epsilon^T) = \Sigma$.

The WLS problem is still that of minimizing $f(\Delta P) = ||W^{1/2}(O_{mc} - A_p \cdot \Delta P)||^2$ and the solution as before, is,

$$\Delta P'' = \Delta P' = \left(A_p^T W A_p\right)^{-1} A_p^T W O_{mc} \tag{37}$$

However, the covariance matrix, as would be expected, is affected by the Q-parameter uncertainties. Thus

$$\Delta P'' = \left(A_p^T W A_p\right)^{-1} A_p^T W(A_p \Delta P_t + A_q \Delta Q_t + \epsilon)$$

or

$$\delta P'' = \Delta P'' - \Delta P_t = \left(A_p^T W A_p\right)^{-1} A_p^T W(A_q \Delta Q_t + \epsilon) \tag{38}$$

It is seen immediately that if $E(\Delta Q_t) = 0$ (meaning that unbiased estimates $Q_o$ of the Q parameters are being used) and $E(\epsilon) = 0$, then $E(\delta P'') = 0$ and $\Delta P''$ is an unbiased estimate of $\Delta P_t$. The covariance matrix, $C(P'') = E(\delta P'' \, \delta P''^T)$, is, by forming the indicated product, choosing $W = \Sigma^{-1}$ (requiring uncorrelated observational errors) and taking the expected value,

$$C(P'') = \left(A_p^T W A_p\right)^{-1} + \left(A_p^T W A_p\right)^{-1} A_p^T W A_q C(Q) A_q^T W A_p \left(A_p^T W A_p\right)^{-1} \tag{39}$$

where $C(Q) = E(\Delta Q_t \Delta Q_t^T)$ is the covariance matrix of the Q parameters.

From the formula, we see that the uncertainty in the estimate $P'' = P_o + \Delta P''$, as represented by the covariance matrix $C(P'')$, is in two parts. The first term is $C(P') = (A_p^T W A_p)^{-1}$, the covariance matrix (or uncertainty) in the estimate $P''$ resulting from random noise in the observations. The second term contains the additional uncertainties in the estimate of the P parameters for having used uncertain values of the Q parameters in the process. Obviously $C(P'')$ reduces to $C(P')$ for $C(Q) = 0$.

The effect upon the estimate $P'' = P_o + \Delta P''$ of an error (as opposed to an uncertainty) in a Q parameter can also be predicted. The estimate $P''(Q_o)$ of P using the value $O_o$ is

$$P''(Q_o) = P_o + \Delta P''(Q_o) = P_o + \left(A_p^T W A_p\right)^{-1} A_p^T W O_{mc}(Q_o) \tag{40}$$

and similarly, using an "erroneous" or alternate value $Q_1$

$$\begin{aligned} P''(Q_1) = P_o + \Delta P''(Q_1) &= P_o + \left(A_p^T W A_p\right)^{-1} A_p^T W O_{mc}(Q_1) \\ &= P_o + \left(A_p^T W A_p\right)^{-1} A_p^T W \left[O_{mc}(Q_o) + \frac{\partial O_{mc}}{\partial Q_o}(Q_1 - Q_o)\right] \end{aligned} \tag{41}$$

Then the difference in the estimates is

$$P''(Q_1) - P''(Q_o) = \left[\left(A_p^T W A_p\right)^{-1} A_p^T W \frac{\partial O_{mc}}{\partial O_o}\right](O_1 - Q_o) \tag{42}$$

Since $O_c$ enters negatively into $O_{mc}(O_{mc} = O_m - O_c)$ and since $\frac{\partial O_c}{\partial Q_o} = A_q$, we have

$$P''(Q_1) - P''(Q_o) = \left[-\left(A_p^T W A_p\right)^{-1} A_p^T W A_q\right](Q_1 - Q_o) \tag{43}$$

The content of the square brackets is evidently the matrix of partial derivatives $\frac{\partial P''}{\partial Q_o}$.

Now $C(P'')$ can be rewritten as

$$C(P'') = C(P') + \left(\frac{\partial P''}{\partial Q_o}\right) C(Q) \left(\frac{\partial P''}{\partial Q_o}\right)^T \tag{44}$$

### 2.5.4 Parameter Transformations

The covariance matrices $C(P')$ or $C(P'')$ would normally include, roughly speaking, the uncertainties in the orbital parameters due to observational errors and Q-parameter uncertainties. The orbital parameters might be spherical coordinates at time $t_o$, for example. Quite evidently their uncertainties are not very descriptive of the resulting trajectory, period, observational, or other related uncertainties; hence the need for transforming the basic covariance matrices to other coordinate systems and reference times. A common requirement, for example, is the covariance matrix of satellite position and velocity at time t, resolved into radial, in-track and cross-track components.

Suppose that a set of parameters $X(t)$, intentionally suggesting $X = (x, y, z, \dot{x}, \dot{y}, \dot{z})^T$ at time $t \neq t_o$, is related to our P and O parameters, and that

$$\delta X = \frac{\partial X}{\partial P_o} \delta P + \frac{\partial X}{\partial Q_o} \delta Q \tag{45}$$

This relation gives the first-order effect upon $X(P_o, Q_o, t)$ of variations $\delta P$ and $\delta Q$ from the nominal values $P_o$ and $Q_o$. If $P_o$ is an unbiased estimate $P''$ of the true vector $P_t$, so that $E(\delta P'') = 0$, then the uncertainty in $X(t)$, due to random errors and Q-parameter uncertainties, can be written

$$\begin{aligned}
\delta X &= \frac{\partial X}{\partial P_o}\,\delta P'' + \frac{\partial X}{\partial Q_o}\,\delta Q \\
&= \frac{\partial X}{\partial P_o}\left(A_p^T W A_p\right)^{-1} A_p^T W (A_q \delta Q + \epsilon) + \frac{\partial X}{\partial Q_o}\,\delta Q \\
&= \left(-\frac{\partial X}{\partial P_o}\,\frac{\partial P''}{\partial Q_o} + \frac{\partial X}{\partial Q_o}\right)\delta Q + \frac{\partial X}{\partial P_o}\left(A_p^T W A_p\right)^{-1} A_p^T W \epsilon \qquad (46)
\end{aligned}$$

Since we are assuming that $E(\epsilon) = E(\delta Q) = 0$ (the latter meaning that $Q_o$ is an unbiased estimate of Q), and that $\epsilon$ and $\delta Q$ are uncorrelated random variables, we see that $E(\delta X) = 0$ and that the covariance matrix $C(X(t))$ is $E(\delta X\, \delta X^T)$ or

$$C(X(t)) = \left(\frac{\partial X}{\partial P_o}\right) C(P')\left(\frac{\partial X}{\partial P_o}\right)^T + \left(\frac{\partial X}{\partial Q_o} - \frac{\partial X}{\partial P_o}\,\frac{\partial P''}{\partial Q_o}\right) C(Q) \left(\frac{\partial X}{\partial Q_o} - \frac{\partial X}{\partial P_o}\,\frac{\partial P''}{\partial Q_o}\right)^T \qquad (47)$$

This is the general formula for transforming P- and Q-parameter covariance matrices into the covariance matrix for any set of related parameters. Using the suggested interpretation $X = (x, y, z, \dot{x}, \dot{y}, \dot{z})^T$, the partial derivates $\frac{\partial X}{\partial P_o}$ and $\frac{\partial X}{\partial Q_o}$ are either just the solutions of the variational equations (insofar as $P_o$ and $Q_o$ represent trajectory parameters), or are computed from geometrical relations (in the case of observational parameters). For other sets of parameters at time t, such as the spherical coordinates $R(t) = (\alpha, \delta, \beta, A, r, v)^T$, the partial derivative matrices $\frac{\partial R}{\partial P_o}$ and $\frac{\partial R}{\partial Q_o}$ can be computed as $\frac{\partial R}{\partial P_o} = \frac{\partial R}{\partial X}\,\frac{\partial X}{\partial P_o}$ (and similarly for $\frac{\partial R}{\partial Q_o}$), wherein

$\frac{\partial R}{\partial X}$ is computed simply from the equations, such as $r = (x^2 + y^2 + z^2)^{1/2}$, which relate the spherical and cartesian coordinates at any time. In practice, the covariance matrix is most easily obtained from $C(X(t))$ by

$$C(R(t)) = \left(\frac{\partial R}{\partial X}\right) C(X(t)) \left(\frac{\partial R}{\partial X}\right)^T \tag{48}$$

SECTION 3

# EQUATIONS AND METHODS

## 3.1 COORDINATE SYSTEMS

There are two coordinate systems employed in TRACE. The earth-centered inertial system, known as the "mean equator and equinox of date," is basic to all the computations, and position and velocity in this system may be expressed in one of three types of coordinates (paragraph 3.1.1). A station-dependent system has also been introduced to facilitate computations involving radar observations and data studies (paragraph 3.1.2).

### 3.1.1 Earth-Centered Inertial System

The basic coordinate system is as follows:

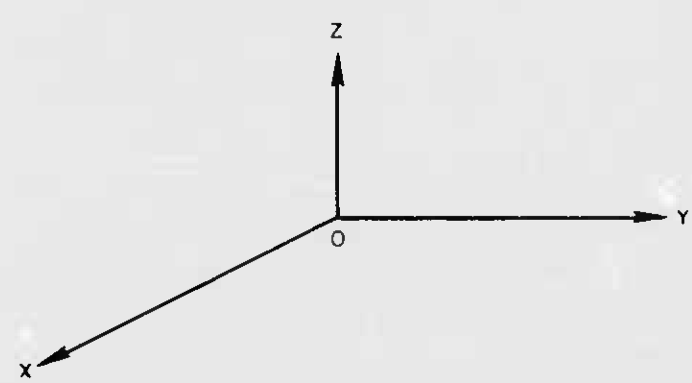



Figure 2. Earth-Centered Coordinate System

where

- 0 is the center of the earth
- X is a vector from 0 in the equatorial plane directed to the vernal equinox at $t_g$, 0 hour GMT of launch date
- Y is a vector from 0 perpendicular to X in such a direction that (X, Y, Z) is a right-handed system
- Z is a vector from 0 perpendicular to the equatorial plane and directed north.

The position and velocity of a body at a point P may be expressed in rectangular or spherical coordinates, or in terms of the classical (elliptic) elements of its orbit, as shown in the following three paragraphs, respectively.

3.1.1.1 Rectangular Coordinates

$P = P(x, y, z, \dot{x}, \dot{y}, \dot{z})$ where x, y, z are the components of position of the body in the X, Y, Z directions, respectively, and $\dot{x}$, $\dot{y}$, $\dot{z}$ are the components of its velocity in these directions.

3.1.1.2 Spherical Coordinates

$P = P(\alpha, \delta, \beta, A, r, v)$

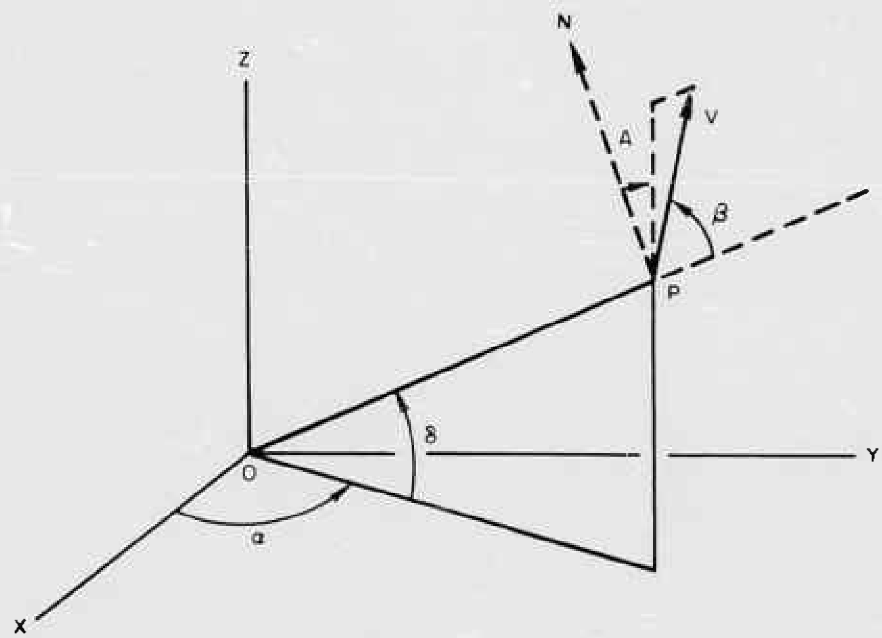



Figure 3. Spherical Coordinates

where

V = a vector equal in magnitude and direction to the velocity of the vehicle at P

α = right ascension measured from the X-axis, positive eastward

δ = geocentric latitude

β = angle between V and the geocentric vertical at P

A = azimuth of V from true north measured on a plane normal to the geocentric vertical

r = magnitude of $\overline{OP}$

v = magnitude of V.

3.1.1.3 Orbital Elements

$P = P(a, e, i, \Omega, \omega, \tau)$

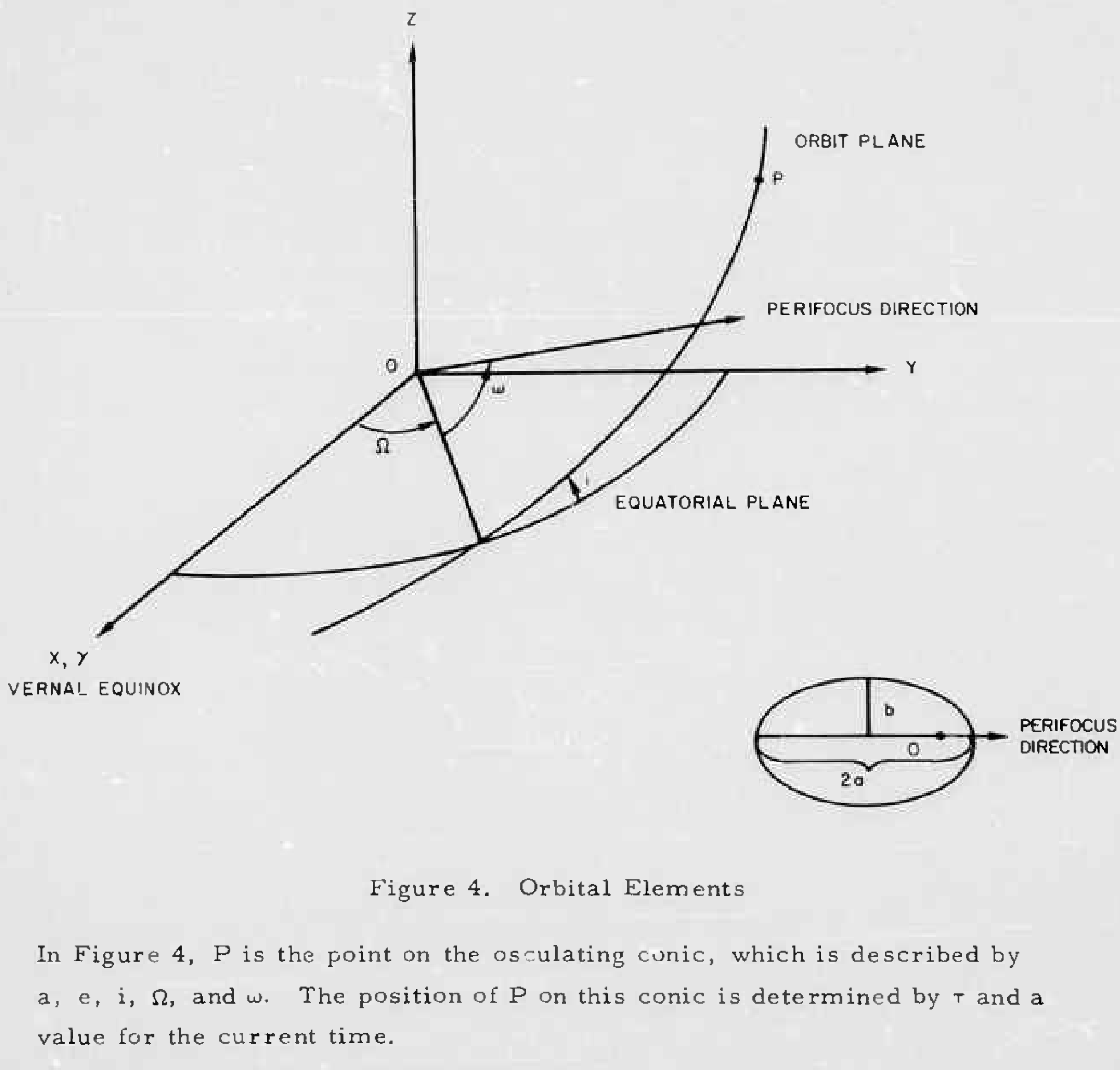



Figure 4. Orbital Elements

In Figure 4, P is the point on the osculating conic, which is described by a, e, i, $\Omega$, and $\omega$. The position of P on this conic is determined by $\tau$ and a value for the current time.

$a$ = semi-major axis

$e$ = eccentricity = $\sqrt{a^2 - b^2}/a$ ($b$ = semi-minor axis)

$i$ = inclination of the orbit plane

$\Omega$ = right ascension of the ascending node

$\omega$ = angle between the direction of perigee and the line of nodes

$\tau$ = time in minutes from $t_g$ of last perigee passage.

3.1.2 Station-Dependent System

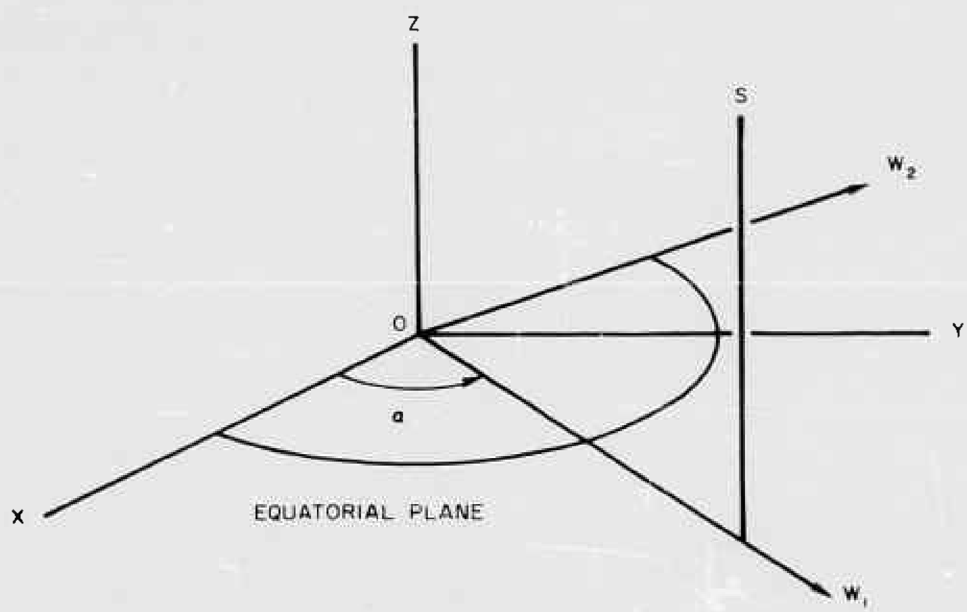



Figure 5. Station Coordinate System

where:

$S$ = the location of the station at some time $t$

$a = \ell + a_g + \omega_e(t - t_g)$

$\ell$ = the geographic longitude of the station

$a_g$ = the right ascension of Greenwich at time $t_g$

$\omega_e$ = the rate of rotation of the earth

$W_1$, $W_2$ = the axes X and Y, rotated through the angle $a$.

## 3.2 INITIAL CONDITIONS

The parameters of the orbit may be input in any of the coordinates described in paragraph 3.1.1. The trajectory computations require earth-centered inertial coordinates; the output includes spherical coordinates and elements. The formulae for the necessary transformations follow. The date chosen to determine the X-axis is $t_g$, zero hour GMT of launch date; the time $t_o$ at which the parameters are specified is with reference to this date.

### 3.2.1 Spherical to Rectangular

$$x = r \cos \delta \cos \alpha$$

$$y = r \cos \delta \sin \alpha$$

$$z = r \sin \delta$$

$$\dot{x} = v \left[\cos \alpha \left(-\cos A \sin \beta \sin \delta + \cos \beta \cos \delta\right) - \sin A \sin \beta \sin \alpha\right]$$

$$\dot{y} = v \left[\sin \alpha \left(-\cos A \sin \beta \sin \delta + \cos \beta \cos \delta\right) + \sin A \sin \beta \cos \alpha\right]$$

$$\dot{z} = v \left[\cos A \cos \delta \sin \beta + \cos \beta \sin \delta\right]$$

If longitude ($\ell$) is input instead of $\alpha$, $\alpha$ is computed as in paragraph 3.1.2. In this case $t - t_g = t_o$.

### 3.2.2 Rectangular to Spherical

$$\alpha = \tan^{-1}(y/x)$$

$$\delta = \tan^{-1}\left(z/\sqrt{x^2 + y^2}\right)$$

$$\beta = \cos^{-1}\left[(x\dot{x} + y\dot{y} + z\dot{z})/rv\right]$$

$$A = \tan^{-1}\left[\frac{r(x\dot{y} - y\dot{x})}{y(y\dot{z} - z\dot{y}) - x(z\dot{x} - x\dot{z})}\right]$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

3.2.3 Elements to Rectangular

$$x = x_\omega P_x + y_\omega Q_x$$

$$y = x_\omega P_y + y_\omega Q_y$$

$$z = x_\omega P_z + y_\omega Q_z$$

$$\dot{x} = \dot{x}_\omega P_x + \dot{y}_\omega Q_x, \text{ etc.}$$

where:

$$P_x = \cos\Omega \cos\omega - \sin\Omega \sin\omega \cos i$$

$$P_y = \sin\Omega \cos\omega + \cos\Omega \sin\omega \cos i$$

$$P_z = \sin\omega \sin i$$

$$Q_x = -\cos\Omega \sin\omega - \sin\Omega \cos\omega \cos i$$

$$Q_y = -\sin\Omega \sin\omega + \cos\Omega \cos\omega \cos i$$

$$Q_z = \cos\omega \sin i$$

$p = a(1-e^2)$, (semi-latus rectum)

$\mu$ = gravitational constant

$n = \sqrt{|\mu/a^3|}$ = mean motion

$M = n(t - \tau)$ = mean anomaly

$E$ = solution of $(M = E - e \sin E)$ = eccentric anomaly

$$r_\omega = a(1 - e\cos E)$$

$$x_\omega = a(\cos E - e)$$

$$y_\omega = \sqrt{|ap|} \sin E$$

$$\dot{x}_\omega = -\frac{\sqrt{|\mu a|}}{r_\omega} \sin E$$

$$\dot{y}_\omega = \frac{\sqrt{\mu p}}{r_\omega} \cos E$$

(These formulae are for the ellipse; if the conic is a hyperbola ($e > 1$), E is the solution of $M = e \sinh E - E$; and $\sin E$ and $\cos E$ above are replaced by $\sinh E$ and $\cosh E$.)

3.2.4 Rectangular to Elements

$$a = \left(\frac{2}{r} - \frac{v^2}{\mu}\right)^{-1}$$

$$e = \sqrt{(e \cos E)^2 + (e \sin E)^2}$$

$$= \sqrt{(e \cosh E)^2 - (e \sinh E)^2} \qquad \text{for hyperbolic orbits}$$

$$i = \tan^{-1}\left(\frac{\sqrt{P_z^2 + Q_z^2}}{P_x Q_y - P_y Q_x}\right)$$

$$\Omega = \tan^{-1}\left(\frac{P_y Q_z - P_z Q_y}{P_x Q_z - P_z Q_x}\right)$$

$$\omega = \tan^{-1}\left(\frac{P_z}{Q_z}\right)$$

$$\tau = t - \frac{M}{n}$$

where:

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$$

$$e \cos E = 1 - \frac{r}{a} \qquad \left( \left. \begin{array}{l} e \cosh E = \\ \\ e \sinh E = \end{array} \right\} \text{for the hyperbola} \right)$$

$$e \sin E = \frac{x\dot{x} + y\dot{y} + z\dot{z}}{\sqrt{|a|\mu}}$$

$$p = \frac{r^2 v^2 - (x\dot{x} + y\dot{y} + z\dot{z})^2}{\mu}$$

$$D = \frac{x\dot{x} + y\dot{y} + z\dot{z}}{e\mu}$$

$$\dot{D} = \frac{e \cos E}{er}$$

$$H = \frac{1}{e\sqrt{\mu p}} \quad (r - p)$$

$$\dot{H} = \frac{1}{e\sqrt{\mu p}} \quad \frac{x\dot{x} + y\dot{y} + z\dot{z}}{r}$$

$$P_x = \dot{D}x - D\dot{x}$$

$$P_y = \dot{D}y - D\dot{y}$$

$$P_z = \dot{D}z - D\dot{z}$$

$$Q_x = \dot{H}x - H\dot{x}, \text{ etc.}$$

$$n = \sqrt{\left|\frac{\mu}{a^3}\right|}$$

$$M = E - e \sin E \text{ or } e \sinh E - E$$

$$E = \tan^{-1}\left(\frac{e \sin E}{e \cos E}\right)$$

3.3 RADAR DATA

All radar observation input is converted to the basic set: R, A, E, $\dot{R}$, $\dot{P}$, $\dot{Q}$, P, Q. This section contains the formulae for these conversions. Input is generally in feet, degrees, and seconds, while internal units are earth-radii, radians, and minutes. The details of units conversion have been omitted.

First, however, Figures 6 through 9 depict most of the radar quantities in Sections 3.3 and 3.4.

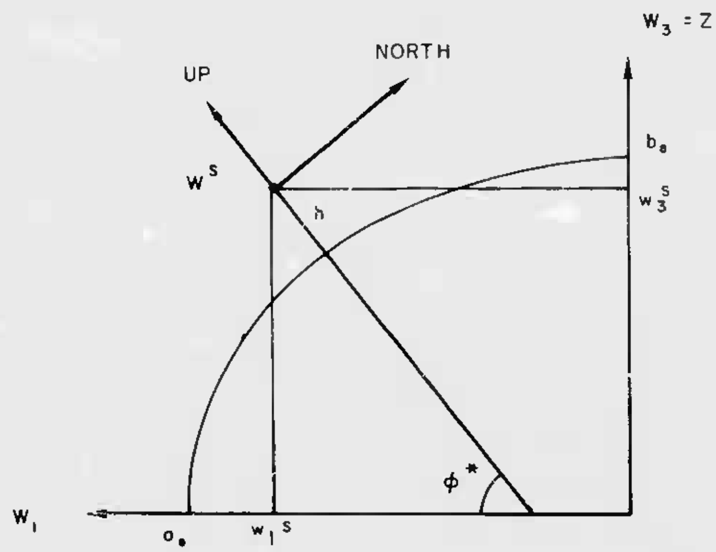



Figure 6. Radar Station Coordinates

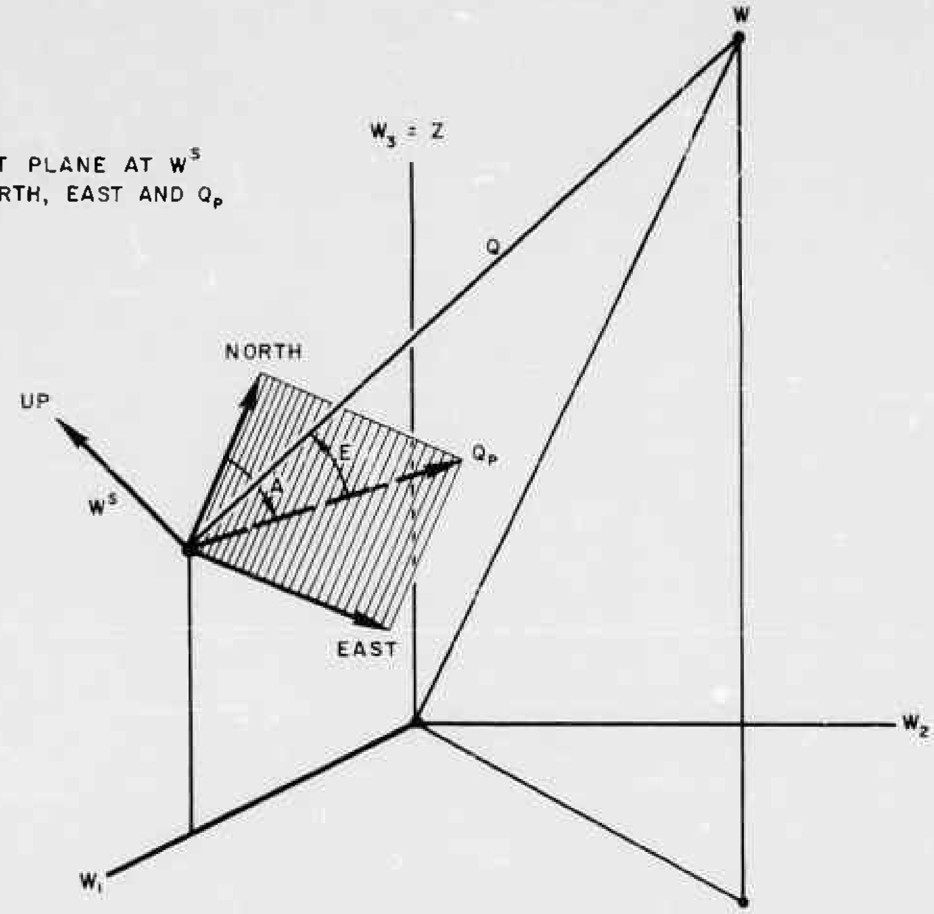


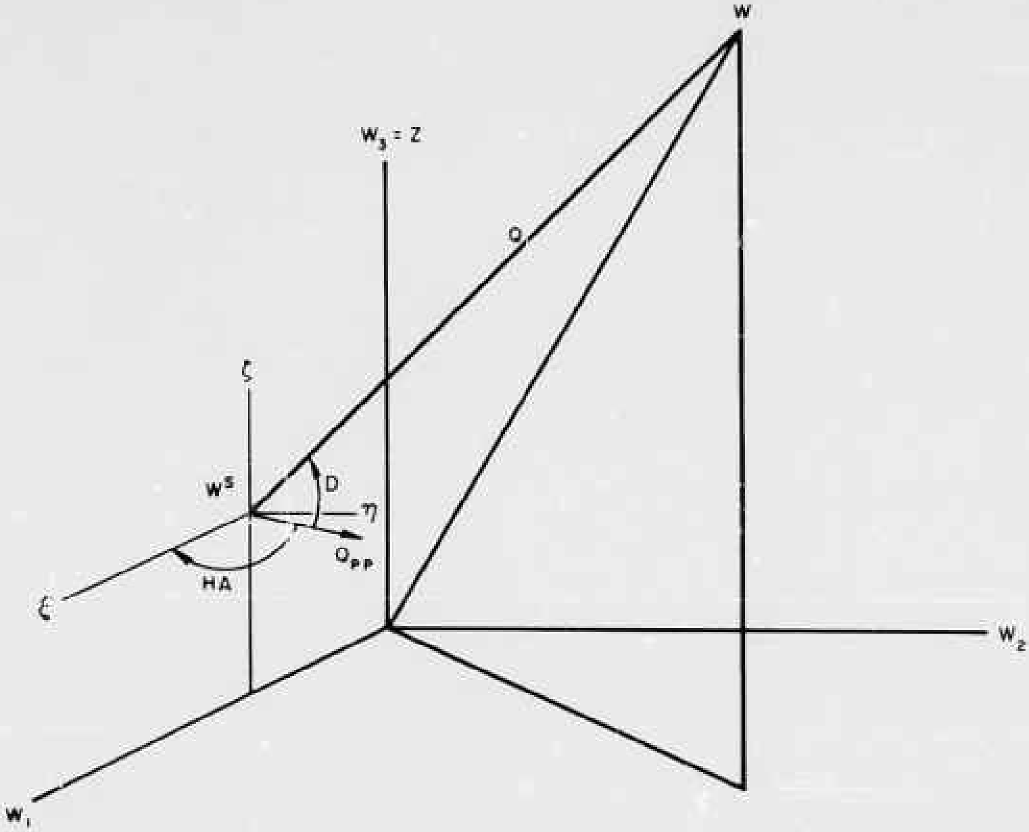

Figure 7. Azimuth and Elevation in Station Coordinate System

$W$ is position of the vehicle

$W^S$ is position of the station

$Q_p$ is the projection of $Q = W - W^S$ onto the tangent plane at $W^S$

$R = |Q|$




HERE $\xi, \eta, \zeta$ ARE PARALLEL TO $W_1, W_2, W_3$, RESPECTIVELY
$Q_{PP}$ IS PROJECTION OF Q ONTO $\xi, \eta$ PLANE
D = TOPOCENTRIC OECLINATION
HA = TOPOCENTRIC HOUR ANGLE

Figure 8. Hour Angle and Declination in Station Coordinate System

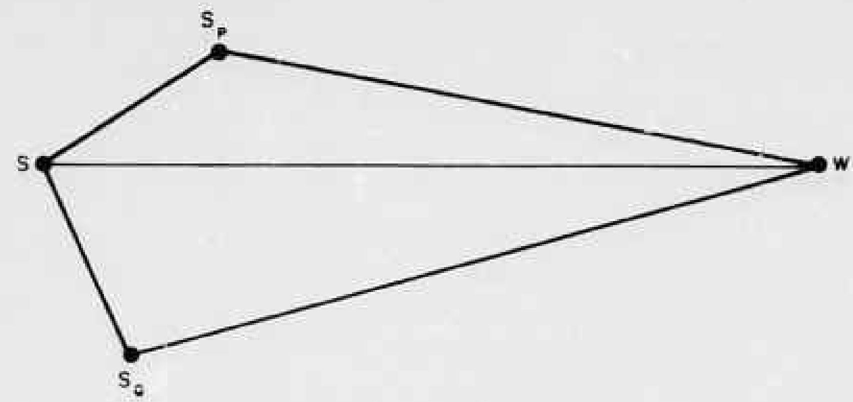



Figure 9. Station Network for Interferometer Data

Here S, $S_p$, and $S_Q$ are a network of stations that report range and range rate differences. Let

$$R = |W-S|\ ,\ R_p = |W-S_p|\ ,\ R_Q = |W-S_Q|$$

Then

$$P = R - R_P$$

$$Q = R - R_Q$$

$$\dot{P} = \dot{R} - \dot{R}_P$$

$$\dot{Q} = \dot{R} - \dot{R}_Q$$

3.3.1 Hour Angle, Declination (HA, D) to A, E

$$E = \sin^{-1} (\sin \Phi^* \sin D + \cos \Phi^* \cos D \cos HA)$$

$$A = \tan^{-1} \left( \frac{\sin HA \cos D}{\cos \Phi^* \sin D - \sin \Phi^* \cos HA \cos D} \right)$$

where $\Phi^*$ = geodetic latitude of the station.

3.3.2 Right Ascension, Declination (RA, D) to A, E

HA = $\alpha$ - RA; compute $\alpha$ as in paragraph 3.1.2, and then apply paragraph 3.3.1.

3.3.3 $\Delta f$, $\Delta t$, to $\dot{R}$, R

$\dot{R} = k_1 \Delta f$ where $k_1$ is input

$R = k_2 \Delta t$ where $k_2$ is input

3.3.4 L, M, N to A, E

$$A = \tan^{-1}\left(\frac{L}{M}\right).$$

$$E = \cos^{-1}\left(\sqrt{L^2 + M^2}\right)$$

3.3.5 $L_1$, $L_2$, $L_3$ to A, E

$$L = c_{12}L_1 + c_{11}L_2$$

$$M = c_{11}L_1 + c_{21}L_2$$

$$N = L_3$$

where

$c_{11} = \cos\psi$, $c_{12} = -\sin\psi$, $c_{21} = \sin\psi$

$\psi$ = rotation angle (input)

and apply paragraph 3.3.4

## 3.4 PARTIAL DERIVATIVES OF RADAR DATA

In tracking and data studies, it is necessary to compute partial derivatives of radar data with respect to parameters of the initial conditions, differential equations, station locations, and observations. For the purposes of this section, it will be assumed that the (integrated) position of the vehicle is known, in earth-centered inertial rectangular coordinates, as are the partial derivatives of these coordinates with respect to the first two types of parameters.

### 3.4.1 Notation

| | |
|---|---|
| $p_i$, $i = 1, 2, \ldots, n$ | the ordered list of initial condition and differential equation parameters for which partials are to be computed |
| $\frac{\partial x}{\partial p_i}, \frac{\partial y}{\partial p_i}, \ldots, \frac{\partial \dot{z}}{\partial p_i}$ | the partial derivatives of $x, y, \ldots, \dot{z}$ with respect to the $p_i$ |
| $\ell$ | longitude of the station |
| $\Phi^*$ | geodetic latitude of the station |
| $h$ | height of the station |
| $\alpha$ | $\alpha_g + \omega_e(t - t_g) + \ell$, as in |
| $w_j$, $\dot{w}_j$, $j = 1, 2, 3$ | position and velocity of the vehicle in the station-dependent W-system |
| $w_1^s$, $w_3^s$ | position of the station in the above system |
| $\epsilon$ | ellipticity of the reference ellipsoid |
| $a_e$ | semi-major axis of the earth |
| $b_e = a_e(1-\epsilon)$ | semi-minor axis of the earth. |

3.4.2 <u>Position and Velocity in the W-system, and Associated Preliminary Computations</u>

$$w_1 = x \cos \alpha + y \sin \alpha$$

$$w_2 = -x \sin \alpha + y \cos \alpha$$

$$w_3 = z$$

$$\dot{w}_1 = (\dot{x} + \omega_e y) \cos \alpha + (\dot{y} - \omega_e x) \sin \alpha$$

$$\dot{w}_2 = -(\dot{x} + \omega_e y) \sin \alpha + (\dot{y} - \omega_e x) \cos \alpha$$

$$\dot{w}_3 = \dot{z}$$

To transform the partial derivatives of the Earth-Centered Inertial (ECI) rectangular coordinates to the station-dependent system, the following quantities are necessary.

Differentiation of the above six equations shows that a simple substitution of

$$\frac{\partial w_j}{\partial p_i} \text{ for } w_j, \ \frac{\partial \dot{w}_j}{\partial p_i} \text{ for } \dot{w}_j, \ j = 1, 2, 3, \text{ and } \frac{\partial x}{\partial p_i}, \frac{\partial y}{\partial p_i}, \ldots, \frac{\partial \dot{z}}{\partial p_i}$$

for $x, y, \ldots, \dot{z}$ yields

$$\frac{\partial w_1}{\partial p_i} = \frac{\partial x}{\partial p_i} \cos \alpha + \frac{\partial y}{\partial p_i} \sin \alpha$$

$$\frac{\partial w_2}{\partial p_i} = \frac{-\partial x}{\partial p_i} \sin \alpha + \frac{\partial y}{\partial p_i} \cos \alpha, \text{ etc.}$$

Differentiating with respect to $\ell$, since $\frac{\partial \alpha}{\partial \ell} = 1$, gives:

$$\frac{\partial w_1}{\partial \ell} = -x \sin \alpha + y \cos \alpha = w_2$$

$$\frac{\partial w_2}{\partial \ell} = -x \cos \alpha - y \sin \alpha = -w_1$$

$$\frac{\partial \dot{w}_1}{\partial \ell} = \dot{w}_2$$

$$\frac{\partial \dot{w}_2}{\partial \ell} = -\dot{w}_1$$

To find the station position in the W-system, we use:

$$w_1^s = (a_e A_s + h) \cos \Phi^*$$

$$w_3^s = (b_e B_s + h) \sin \Phi^*$$

where:

$$A_s = (\cos^2 \Phi^* + \frac{b_e^2}{a_e^2} \sin^2 \Phi^*)^{-1/2}$$

$$B_s = (\sin^2 \Phi^* + \frac{a_e^2}{b_e^2} \cos^2 \Phi^*)^{-1/2}$$

Differentiating with respect to $\Phi^*$ and h,

$$\frac{\partial w_1^s}{\partial \Phi^*} = -w_3^s \quad (1)$$

$$\frac{\partial w_3^s}{\partial \Phi^*} = w_1^s \quad (1)$$

(1) Approximate formulas; correct only for a spherical earth.

$$\frac{\partial w_1^s}{\partial h} = \cos \Phi^*$$

$$\frac{\partial w_3^s}{\partial h} = \sin \Phi^*$$

At this point it is convenient to introduce three intermediate vectors Q, U, and V, and the quantity $R_1$.

$Q = W - W^s$ vehicle position relative to the station.

$$q_1 = w_1 - w_1^s$$

$$q_2 = w_2$$

$$q_3 = w_3 - w_3^s$$

$$R = |Q| = \sqrt{q_1^2 + q_2^2 + q_3^2}$$

$U = Q/R$, a unit vector in the direction of Q

$$u_1 = q_1/R$$

$$u_2 = q_2/R$$

$$u_3 = q_3/R$$

V is the vector U referred to the East-North-Up system

$$v_1 = u_2$$

$$v_2 = -u_1 \sin \Phi^* + u_3 \cos \Phi^*$$

$$v_3 = u_1 \cos \Phi^* + u_3 \sin \Phi^*$$

$$v = \sqrt{v_1^2 + v_2^2}$$

$$R_1 = vR$$

Then,

$$v_3 = \sin E$$

$$v = \cos E$$

$$\frac{v_2}{v} = \cos A$$

$$\frac{v_1}{v} = \sin A$$

To compute the $\frac{\partial A}{\partial t}$ and $\frac{\partial E}{\partial t}$ we will need $\dot{v}$, and to this end we compute

$$\dot{U} = \frac{1}{R} [\dot{W} - U\dot{R}] = \frac{1}{R} [\dot{W} - (U \cdot \dot{W})\, U]$$

then $\dot{v}_1 = \dot{u}_2$

$$\dot{v}_2 = -\dot{u}_1 \sin \Phi^* + \dot{u}_3 \cos \Phi^*$$

$$\dot{v} = \frac{v_1 \dot{v}_1 + v_2 \dot{v}_2}{v}$$

### 3.4.3 Range Partials

Differentiating $R = \sqrt{q_1^2 + q_2^2 + q_3^2}$ results in:

$$\frac{\partial R}{\partial p_i} = U \cdot \frac{\partial Q}{\partial p_i} = u_1 \frac{\partial q_1}{\partial p_i} + u_2 \frac{\partial q_2}{\partial p_i} + u_3 \frac{\partial q_3}{\partial p_i}$$

$$\frac{\partial R}{\partial \Phi^*} = u_1 w_3^s - u_3 w_1^s$$

$$\frac{\partial R}{\partial \ell} = u_1 w_2 - u_2 w_1$$

$$\frac{\partial R}{\partial h} = -u_1 \cos \Phi^* - u_3 \sin \Phi^*$$

$$\frac{\partial R}{\partial R_{bias}} = 1$$

$$\frac{\partial R}{\partial t} = \dot{R} = (U \cdot \dot{W})$$

### 3.4.4 Azimuth Partials

Differentiating $A = \tan^{-1} (v_1/v_2)$;

$$\frac{\partial A}{\partial p_i} = \frac{1}{R_1} \left[ \frac{\partial w_2}{\partial p_i} \cos A - \left( - \frac{\partial w_1}{\partial p_i} \sin \Phi^* + \frac{\partial w_3}{\partial p_i} \cos \Phi^* \right) \sin A \right]$$

$$\frac{\partial A}{\partial \Phi^*} = \frac{\sin A}{R_1} \left( w_1 \cos \Phi^* + w_3 \sin \Phi^* \right)$$

$$\frac{\partial A}{\partial \ell} = \frac{-w_1 \cos A + w_2 \sin \Phi^* \sin A}{R_1}$$

$$\frac{\partial A}{\partial h} = 0$$

$$\frac{\partial A}{\partial A_{bias}} = 1$$

$$\frac{\partial A}{\partial t} = \frac{1}{v^2} \left( v_2 \dot{v}_1 - v_1 \dot{v}_2 \right)$$

3.4.5 Elevation Partials

Differentiating $E = \sin^{-1} v_3 = \cos^{-1} v$,

$$\frac{\partial E}{\partial p_i} = \frac{1}{R_1} \left( \frac{\partial w_1}{\partial p_i} \cos \Phi^* + \frac{\partial w_3}{\partial p_i} \sin \Phi^* - \frac{\partial R}{\partial p_i} \sin E \right)$$

$$\frac{\partial E}{\partial \Phi^*} = \frac{1}{R_1} \left( w_3 \cos \Phi^* - w_1 \sin \Phi^* - \frac{\partial R}{\partial \Phi^*} \sin E \right)$$

$$\frac{\partial E}{\partial \ell} = \frac{1}{R_1} \left( w_2 \cos \Phi^* - \frac{\partial R}{\partial \ell} \sin E \right)$$

$$\frac{\partial E}{\partial h} = \frac{-1}{R_1} \left( 1 + \frac{\partial R}{\partial h} \sin E \right)$$

$$\frac{\partial E}{\partial E_{bias}} = 1$$

$$\frac{\partial E}{\partial t} = \frac{\dot{u}_1 \cos \Phi^* + \dot{u}_3 \sin \Phi^*}{\cos E}$$

3.4.6 Range Rate Partials

Differentiating $\dot{R} = (U \cdot \dot{W})$,

$$\frac{\partial \dot{R}}{\partial p_i} = \left( \frac{\partial W}{\partial p_i} \cdot \dot{U} \right) + \left( U \cdot \frac{\partial \dot{W}}{\partial p_i} \right)$$

$$\frac{\partial \dot{R}}{\partial \Phi^*} = w_3^s \dot{u}_1 - w_1^s \dot{u}_3$$

$$\frac{\partial \dot{R}}{\partial \ell} = (w_2 \dot{u}_1 - w_1 \dot{u}_2) + (\dot{w}_2 u_1 - \dot{w}_1 u_2)$$

$$\frac{\partial \dot{R}}{\partial h} = - \dot{u}_1 \cos \Phi^* - \dot{u}_3 \sin \Phi^*$$

$$\frac{\partial \dot{R}}{\partial \dot{R}_{bias}} = 1$$

$$\frac{\partial \dot{R}}{\partial t} = \ddot{R} = (\dot{U} \cdot \dot{W}) + (U \cdot \ddot{W})$$

where $\ddot{W} = - \left( \omega_e^2 J + \frac{\mu}{|X|^3} I \right) W + 2L\dot{X}$

$$J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad L\dot{X} = \begin{pmatrix} -\omega_e \dot{x} \sin \alpha + \omega_e \dot{y} \cos \alpha \\ -\omega_e \dot{x} \cos \alpha - \omega_e \dot{y} \sin \alpha \\ 0 \end{pmatrix}$$

$$|X| = \left( \sum_{i=1}^{3} w_i^2 \right)^{1/2}$$

3.4.7 P, Q, $\dot{P}$, $\dot{Q}$, Partials

These partial derivatives are obtained by differencing the R, $\dot{R}$ partials using the appropriate station locations.

### 3.4.8 $\dot{A}$, $\dot{E}$ Partials

$$\frac{\partial \dot{A}}{\partial p_i} = \frac{1}{Rv} \left\{ \cos A \left[ \frac{\partial \dot{w}_2}{\partial p_i} + \dot{A} \left( \frac{\partial w_1}{\partial p_i} \sin \Phi^* - \frac{\partial w_3}{\partial p_i} \cos \Phi^* \right) \right] + \sin A \right.$$

$$\left. \left[ \left( \frac{\partial \dot{w}_1}{\partial p_i} \sin \Phi^* - \frac{\partial \dot{w}_3}{\partial p_i} \cos \Phi^* \right) - \dot{A} \frac{\partial w_2}{\partial p_i} \right] - (\dot{R}v + R\dot{v}) \frac{\partial A}{\partial p_i} \right\}$$

$$\frac{\partial \dot{E}}{\partial p_i} = \frac{1}{Rv} \left[ \frac{\partial \dot{w}_1}{\partial p_i} \cos \Phi^* + \frac{\partial \dot{w}_3}{\partial p_i} \sin \Phi^* - \frac{\partial \dot{R}}{\partial p_i} \sin E - \dot{E} \frac{\partial R}{\partial p_i} \cos E \right.$$

$$\left. - (\dot{R}v + R\dot{v}) \frac{\partial E}{\partial p_i} \right]$$

## 3.5 DATA GENERATION CALCULATIONS

The formulae used to compute data for data generation are a subset of those used in Sections 3.1 and 3.4 with three exceptions.

### 3.5.1 Rise-Set Prediction

$$\underline{r} \cdot \underline{R} - rR \cos\left(\frac{\pi}{2} - E_m - \sin^{-1} \frac{R \cos E_m}{r}\right) = 0$$

where

$\underline{r}$ = vehicle position vector

$\underline{R}$ = station position vector

$E$ = elevation

$E_m$ = input minimum elevation or input maximum elevation (whichever is applicable).

This equality holds when the elevation $E = E_m$ in a two-body model. The equation is positive when $E > E_m$ and negative when $E < E_m$. Preliminary values of rise-set times are generated by converting the above equation to a function of eccentric anomaly, $\theta$, stepping from $\theta_o$ to $\theta_o + 2\pi$, and noting the times of the appropriate sign changes.

The equations in paragraph 3.5.2 are used to compute the actual rise-set times from the integrated trajectory.

### 3.5.2 Rise and Set Times

$$t \text{ (rise or set)} = t_n + \Delta t$$

$$\Delta t = - \frac{v_3 - \sin(E_m)}{\dot{u}_1 \cos \Phi^* + \dot{u}_3 \sin \Phi^*}$$

$t_n$ = current time

$v_3$ and $\dot{u}_i$ are defined in 3.4.2.

### 3.5.3 Observations with Normally Distributed Random Noise

| | |
|---|---|
| $o = o_c + r_n$ | o is the noisy observation (of type j from station s) |
| | $o_c$ is the nominal computed observation |
| | $r_n$ is the noise added. |
| $r_n = n\,\sigma_{sj} + \beta_{sj}$ | $\sigma_{sj}$ is the appropriate sigma for type j, station s |
| | $\beta_{sj}$ is the appropriate bias (if any) |
| | n is a random element from a set of numbers with mean zero and unity standard deviation. |

### 3.6 TRAJECTORY

The position and velocity components, $X = (x, y, z)$ and $\dot{X} = (\dot{x}, \dot{y}, \dot{z})$, of the vehicle and their partial derivatives, $X_{p_i}$ and $\dot{X}_{p_i}$ $(i = 1, \ldots, n)$, with respect to the trajectory (initial condition and differential equation) parameters are functions of time defined by their differential equations and appropriate initial conditions (paragraphs 3.6.1 and 3.6.2). The equations are integrated numerically (paragraph 3.6.3), and at each observation or print time all the quantities, $X$, $\dot{X}$, $X_{p_i}$, and $\dot{X}_{p_i}$ $(i = 1, \ldots, n)$, are obtained by interpolation (paragraph 3.6.4) in the integrated results; from these the computed radar observations and partial derivatives (Sections 3.3 and 3.4) and the trajectory output (paragraph 3.6.5) are computed.

#### 3.6.1 Differential Equations

The equations of motion of the vehicle are

$$\ddot{X} = \frac{-\mu X}{r^3} + F \tag{50}$$

where $\mu$ is the gravitational constant (GM) of the earth, $r = |X| = (x^2 + y^2 + z^2)^{1/2}$, and $F = F_1 + F_2 + F_3$ is the perturbative acceleration due to asphericity of the earth, extra-terrestrial gravitational forces, and atmospheric drag, respectively. The initial conditions $X(t_o)$ and $\dot{X}(t_o)$, if not given directly, are computed from the initial spherical coordinates or elliptic elements. See Section 3.2 for these formulae.

The perturbative acceleration $F_1$ due to the asphericity of the earth is derived from the assumed potential function.

$$U = \frac{\mu}{r}\left[1 - \sum_{n=2}^{5} J_n\left(\frac{a_e}{r}\right)^n P_n(\sin\phi) + \sum_{n=2}^{4}\sum_{m=1}^{n} J_{nm}\left(\frac{a_e}{r}\right)^n P_n^m(\sin\phi)\cos m(\lambda - \lambda_{nm})\right] \tag{51}$$

where

| | |
|---|---|
| $\mu$ | is the product GM of the Newtonian gravitational constant and the mass of the earth |
| $r, \varphi, \lambda$ | are the geocentric distance, geocentric latitude and (east) longitude of a point |
| $a_e$ | is the mean equatorial radius of the earth |
| $J_n, J_{nm}$ | are numerical coefficients |
| $P_n$ | is the Legendre polynomial of the first kind of degree n |
| $P_n^m$ | is the Legendre associated function of the first kind |
| $\lambda_{nm}$ | are longitudes associated with the $J_{nm}$. |

In the local horizontal coordinate system, in which the coordinate axes are directed Up (along the radius vector), East, and North, the force components are

$$g_U = \frac{\partial U}{\partial r}$$

$$= -\frac{\mu}{r^2}\left[1 - \sum_{n=2}^{5} (n+1) J_n \left(\frac{a_e}{r}\right)^n P_n (\sin\varphi)\right.$$

$$\left. + \sum_{n=2}^{4} \sum_{m=1}^{n} (n+1) J_{nm} \left(\frac{a_e}{r}\right)^n P_n^m (\sin\varphi) \cos m (\lambda - \lambda_{nm})\right]$$

$$g_E = \frac{1}{r \cos\varphi} \frac{\partial U}{\partial \lambda}$$

$$= -\frac{\mu}{r^2} \sum_{n=2}^{4} \sum_{m=1}^{n} m J_{nm} \left(\frac{a_e}{r}\right)^n \frac{P_n^m (\sin\varphi)}{\cos\varphi} \sin m (\lambda - \lambda_{nm})$$

$$g_N = \frac{1}{r}\frac{\partial U}{\partial \varphi}$$

$$= -\frac{\mu}{r^2}\left[\sum_{n=2}^{5} J_n \left(\frac{a_e}{r}\right)^n P_n' (\sin\varphi) \cos\varphi\right.$$

$$\left. - \sum_{n=2}^{4}\sum_{m=1}^{n} J_{nm}\left(\frac{a_e}{r}\right)^n P_n^{m'} (\sin\varphi)\cos\varphi \cos m(\lambda - \lambda_{nm})\right]$$

The Legendre functions and their derivatives are computed from the recursion formulas

$$P_n (\sin\varphi) = \frac{-(n-1)\,P_{n-2}(\sin\varphi) + (2n-1)\sin\varphi\, P_{n-1}(\sin\varphi)}{n}$$

$$P_n' (\sin\varphi) = \sin\varphi\, P_{n-1}' (\sin\varphi) + n\, P_{n-1}(\sin\varphi)$$

$$\frac{P_n^m (\sin\varphi)}{\cos\varphi} = \frac{-(n+m-1)\dfrac{P_{n-2}^m (\sin\varphi)}{\cos\varphi} + (2n-1)\sin\varphi \dfrac{P_{n-1}^m (\sin\varphi)}{\cos\varphi}}{n-m}$$

$$\frac{P_m^m (\sin\varphi)}{\cos\varphi} = 1 \cdot 3 \cdot \ldots (2m-1)(\cos\varphi)^{m-1}$$

$$P_n^{m'} (\sin\varphi)\cos\varphi = (n+1)\sin\varphi \frac{P_n^m (\sin\varphi)}{\cos\varphi} - (n-m+1)\frac{P_{n+1}^m (\sin\varphi)}{\cos\varphi}$$

with the initial values

$$P_o (\sin\varphi) = P_1' (\sin\varphi) = 1,$$

$$P_1 (\sin\varphi) = \sin\varphi,$$

$$\frac{P_{m-1}^m (\sin\varphi)}{\cos\varphi} = 0.$$

The force vector in the ECI coordinate system is then:

$$\begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} = \begin{pmatrix} \cos\varphi\cos\alpha & -\sin\alpha & -\sin\varphi\sin\alpha \\ \cos\varphi\sin\alpha & \cos\alpha & -\sin\varphi\sin\alpha \\ \sin\varphi & 0 & \cos\varphi \end{pmatrix} \begin{pmatrix} g_U \\ g_E \\ g_N \end{pmatrix} \tag{52}$$

where $\alpha = \alpha_g + \omega_e (t - t_g)$ is the right ascension.

The gravitational attraction of other bodies contributes

$$F_2 = -\mu \sum_{j=1}^{k} m_j \left( \frac{X - X_j}{|X - X_j|^3} + \frac{X_j}{|X_j|^3} \right) \tag{53}$$

where $m_j$ is the mass, relative to the earth, of the $j^{th}$ body and $X_j$ is the vector position of the $j^{th}$ body, as obtained from the JPL-STL planetary coordinate tapes. For a description of these tapes and their preparation see Reference 6.

Note that the tabular planetary coordinates are with respect to the Mean Equator and Equinox 1950.0 coordinate system, whereas TRACE calculations are referred to 0 hour GMT of start day. The planetary coordinates are transformed to the coordinate system of TRACE before $F_2$ is calculated. The subroutine is described in Reference 7 which in turn refers to Reference 8.

The effect of atmospheric drag is the term

$$F_3 = -\rho \frac{V_A}{2} \left( \frac{C_D A}{W} \right) \dot{X}_A$$

where $\rho$ is the density at height

$$h = r - \frac{a_e(1-\varepsilon)}{\left[1 - (2\varepsilon - \varepsilon^2)\frac{x^2 + y^2}{r^2}\right]^{1/2}}$$

above the oblate earth, and where

$\frac{C_D A}{W}$ is the drag coefficient, (or "ballistic coefficient"),

$\dot{X}_A$ is the vehicle velocity vector relative to the rotating atmosphere. That is,

$$\dot{x}_A = \dot{x} + \omega_e y,$$

$$\dot{y}_A = \dot{y} - \omega_e x,$$

$$\dot{z}_A = \dot{z}, \text{ and}$$

$$V_A = |\dot{X}_A|.$$

The atmospheric density is computed from an atmosphere model (or certain combinations of models) given by References 9, 10, and 11.

### 3.6.2 Trajectory Partial Derivatives

The partial derivatives of vehicle position and velocity with respect to trajectory parameters can be approximated analytically, or can be obtained by a simultaneous numerical integration of the variational equations.

#### 3.6.2.1 Variational Equations

The variational equation for an initial condition parameter $\alpha$ is

$$\ddot{X}_\alpha = \left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right] X_\alpha + \frac{\partial F}{\partial \dot{X}} \dot{X}_\alpha \tag{54}$$

with initial conditions $X_\alpha(t_o) = \left(\frac{\partial X}{\partial \alpha}\right)_{t_o}$, $\dot{X}_\alpha(t_o) = \left(\frac{\partial \dot{X}}{\partial \alpha}\right)_{t_o}$.

For a differential equation parameter $\beta$,

$$\ddot{X}_\beta = \left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right] X_\beta + \frac{\partial F}{\partial \dot{X}} \dot{X}_\beta + \frac{\partial F}{\partial \beta} \tag{55}$$

with $X_\beta(t_o) = \dot{X}_\beta(t_o) = 0$.

Here $X_\alpha = \frac{\partial X}{\partial \alpha}$, $\dot{X}_\alpha = \frac{\partial \dot{X}}{\partial \alpha}$, $X_\beta$, $\dot{X}_\beta$ and $\frac{\partial F}{\partial \beta}$ are all 3-vectors. The contents of the square brackets and $\frac{\partial F}{\partial \dot{X}}$ are 3 x 3 matrices. The system is solved for each parameter, and all the numerical integrations are carried out simultaneously.

In the above equations the principal contributions to $\frac{\partial F}{\partial X}$ stem from the oblateness coefficient $J_2$ and from the dependence of the drag force upon the position of the vehicle. (The latter is important for low-altitude satellites.) Lesser sources are the other-body gravitational forces and the higher order oblateness terms; they are ignored in the calculation of $\frac{\partial F}{\partial X}$.

The matrix in square brackets is calculated as the sum $V + T$ where $V$ derives from the gravitational force including the $J_2$ oblateness term, and $T$ from the drag force. The spherical earth contribution is easily derived:

$$\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) = -\mu\left(r^{-3}\frac{\partial X}{\partial X} - 3r^{-4} X \frac{\partial r}{\partial X}\right) = 3\mu\left(\frac{-1}{3r^3} I + \frac{XX^T}{r^5}\right) \tag{56}$$

since $\frac{\partial r}{\partial X} = \frac{X^T}{r}$. The oblateness component is not so simply obtained. It is, of course, derived from the contribution of the $J_2$ term to the perturbative acceleration $F_1$, which is in turn the gradient of the potential U. The calculation is tedious and only the final matrix V is given here.

$$V = 3\mu \begin{bmatrix} P_{xx} - Q + J_2 a_e^2 (x^2 S - U) & P_{xy} + xy\, J_2 a_e^2 S & P_{xz} + xz\, J_2 a_e^2 T \\ P_{xy} + xy\, J_2 a_e^2 S & P_{yy} - Q + J_2 a_e^2 (y^2 S - U) & P_{yz} + yz\, J_2 a_e^2 T \\ P_{xz} + xz\, J_2 a_e^2 T & P_{yz} + yz\, J_2 a_e^2 T & P_{zz} - Q + J_2 a_e^2 (z^2 T - 3U) \end{bmatrix} \tag{57}$$

where $J_2$ is the principal oblateness coefficient,

$$P_{xy} = \frac{xy}{r^5}, \quad Q = \frac{1}{3r^3}$$

$$S = \frac{5}{2r^7} \left(1 - \frac{7z^2}{r^2}\right)$$

$$T = \frac{5}{2r^7} \left(3 - \frac{7z^2}{r^2}\right)$$

$$U = \frac{1}{2r^5} \left(1 - \frac{5z^2}{r^2}\right)$$

The T matrix, which shows the dependence of the drag force upon the vehicle position, is derived as follows

$$T = \frac{\partial F_3}{\partial X} = -\frac{1}{2}\left(\frac{C_D A}{W}\right) \frac{\partial}{\partial X} \left(\rho V_A \dot{X}_A\right)$$

$$= -\frac{1}{2}\left(\frac{C_D A}{W}\right)\left(V_A \dot{X}_A \frac{\partial \rho}{\partial X} + \rho \dot{X}_A \frac{\partial V_A}{\partial X} + \rho V_A \frac{\partial \dot{X}_A}{\partial X}\right) \tag{58}$$

The derivatives of $\rho$, $V_A$ and $\dot{X}_A$ are:

(a) $\frac{\partial \rho}{\partial X} = \frac{\partial \rho}{\partial h} \frac{\partial h}{\partial X}$

where

$$h = r - \frac{a_e(1 - \epsilon)}{\left[1 - (2\epsilon - \epsilon^2)\,\dfrac{x^2 + y^2}{r^2}\right]^{1/2}}$$

$$\frac{\partial h}{\partial \mathbf{X}} = \left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}, \frac{\partial h}{\partial z}\right)$$

$$\frac{\partial h}{\partial x} = \frac{x}{r}\left\{1 - \frac{a_e\,\epsilon\,(2 - 3\epsilon + \epsilon^2)\,z^2}{[r^2 - (2\epsilon - \epsilon^2)(x^2 + y^2)]^{3/2}}\right\}$$

$$\frac{\partial h}{\partial y} = \frac{y}{r}\left\{1 - \frac{a_e\,\epsilon\,(2 - 3\epsilon + \epsilon^2)\,z^2}{[r^2 - (2\epsilon - \epsilon^2)(x^2 + y^2)]^{3/2}}\right\}$$

$$\frac{\partial h}{\partial z} = \frac{z}{r}\left\{1 + \frac{a_e\,\epsilon\,(2 - 3\epsilon + \epsilon^2)(x^2 + y^2)}{[r^2 - (2\epsilon - \epsilon^2)(x^2 + y^2)]^{3/2}}\right\}$$

and $\frac{\partial \rho}{\partial h}$, the rate of changes of density with altitude, depends upon the model atmosphere, its parameters, and h.

An approximation to $\frac{\partial \rho}{\partial h}$ in the form

$$\frac{\partial \rho}{\partial h} = \rho' \frac{\rho}{h}$$

may be used by specifying a value for $\rho'$ in each of the intervals $0 \leq h < 108$ n mi and $108 \leq h < 378$ n mi. Alternatively (as in Reference 12), $\frac{\partial \rho}{\partial h}$ may be calculated from density expressions, for $76 \leq h < 108$ n mi

$$\rho_1 = 5.606 \times 10^{-12}\left(\frac{76}{h}\right)^{d_1}\left[\frac{108 - h}{32} + 0.85\left(\frac{h - 76}{32}\right)^{4/3} F_{10.7}\right]\left[1 + \frac{h - 76}{153}\left(\frac{1 + \cos\Psi'}{2}\right)^3\right]$$

and for $108 \leq h < 378$ n mi,

$$\rho_2 = \rho_o(h)\ (0.85F_{10.7}) \left[1 + 0.19\ [\exp(0.0102h) - 1.9] \left(\frac{1 + \cos\Psi'}{2}\right)^3\right]$$

where $\log_{10} \rho_o\ (h) = d_2 - 0.00368\ h + 6.363\ \exp[-0.0048h]$.

Differentiating each of these expressions with respect to h, one obtains

$$\frac{\partial \rho_1}{\partial h} = \frac{-d_1\rho}{h} - 5.606 \times 10^{-12}\left(\frac{76}{h}\right)^{d_1}\left[\frac{1}{32} - \left(\frac{1 + \cos\Psi'}{2}\right)^3\left(\frac{184 - 2h}{4896}\right)\right]$$

$$+ 5.606 \times 10^{-12}\left(\frac{76}{h}\right)^{d_1}\ (0.85)F_{10.7}\left(\frac{h - 76}{32}\right)^{1/3}\left[\frac{1}{24} - \left(\frac{1 + \cos\Psi'}{2}\right)^3\left(\frac{532 - 7h}{14688}\right)\right]$$

and

$$\frac{\partial \rho_2}{\partial h} = -\rho\frac{\left[0.00368 + 0.0305424\ \exp[-0.0048\ h]\right]}{0.4342944819} + (0.85)F_{10.7} \times$$

$$\exp\left[2.302585\ (d_2 - 0.00368h + 6.363\ \exp[-0.0048h])\right] \times$$

$$\left[0.001938\left(\frac{1 + \cos\Psi'}{2}\right)^3 \exp[0.0102h]\right]$$

(b) $$\frac{\partial V_A}{\partial X} = \frac{\partial}{\partial X}\ [(\dot{x} + \omega_e y)^2 + (\dot{y} - \omega_e x)^2 + \dot{z}^2]^{1/2}$$

$$= \frac{\omega_e}{V_A}\ (-\dot{y}_A,\ \dot{x}_A,\ 0).$$

(c) $$\frac{\partial \dot{X}_A}{\partial X} = \begin{pmatrix} 0 & \omega_e & 0 \\ -\omega_e & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Combining these results, the elements $T_{ij}$ of the T matrix are

$$T_{11} = -\frac{C_D A}{2W}\left(V_A \dot{x}_A \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial x} - \frac{\rho \omega_e \dot{x}_A \dot{y}_A}{V_A}\right)$$

$$T_{12} = -\frac{C_D A}{2W}\left(V_A \dot{x}_A \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial y} + \frac{\rho \omega_e \dot{x}_A^2}{V_A} + \rho V_A \omega_e\right)$$

$$T_{13} = -\frac{C_D A}{2W}\left(V_A \dot{x}_A \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial z}\right)$$

$$T_{21} = -\frac{C_D A}{2W}\left(V_A \dot{y}_A \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial x} - \frac{\rho \omega_e \dot{y}_A^2}{V_A} - \rho V_A \omega_e\right)$$

$$T_{22} = -\frac{C_D A}{2W}\left(V_A \dot{y}_A \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial y} + \frac{\rho \omega_e \dot{x}_A \dot{y}_A}{V_A}\right)$$

$$T_{23} = -\frac{C_D A}{2W}\left(V_A \dot{y}_A \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial z}\right)$$

$$T_{31} = -\frac{C_D A}{2W}\left(V_A \dot{z}_A \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial x} - \frac{\rho \omega_e \dot{y}_A \dot{z}_A}{V_A}\right)$$

$$T_{32} = -\frac{C_D A}{2W}\left(V_A \dot{z}_A \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial y} + \frac{\rho \omega_e \dot{x}_A \dot{z}_A}{V_A}\right)$$

$$T_{33} = -\frac{C_D A}{2W}\left(V_A \dot{z}_A \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial z}\right)$$

The matrix $\frac{\partial F}{\partial \dot{X}}$ is simply

$$\frac{\partial F_3}{\partial \dot{X}} = -\frac{1}{2}\rho V_A \frac{C_D A}{W}\left(\frac{\dot{X}_A \dot{X}_A^T}{V_A^2} + I\right). \qquad (59)$$

3.6.2.2 Variational Equation Initial Conditions

The initial conditions, $X_a(t_o)$ and $\dot{X}_a(t_o)$, are given here for three types of parameters.

For parameters in rectangular coordinates of the initial position and velocity,

$$\left(\frac{\partial X}{\partial X}\right)_{t_o} = \left(\frac{\partial \dot{X}}{\partial \dot{X}}\right)_{t_o} = I, \text{ the } 3 \times 3 \text{ identity matrix and}$$

$$\left(\frac{\partial X}{\partial \dot{X}}\right)_{t_o} = \left(\frac{\partial \dot{X}}{\partial X}\right)_{t_o} = 0.$$

For the spherical coordinate parameters,

$\alpha$ (right ascension)

$$\frac{\partial x}{\partial \alpha} = -y \qquad \frac{\partial \dot{x}}{\partial \alpha} = -\dot{y}$$

$$\frac{\partial y}{\partial \alpha} = x \qquad \frac{\partial \dot{y}}{\partial \alpha} = \dot{x}$$

$$\frac{\partial z}{\partial \alpha} = 0 \qquad \frac{\partial \dot{z}}{\partial \alpha} = 0$$

$\delta$ (declination)

$$\frac{\partial x}{\partial \delta} = -r \sin \delta \cos \alpha$$

$$\frac{\partial y}{\partial \delta} = -r \sin \delta \sin \alpha$$

$$\frac{\partial z}{\partial \delta} = r \cos \delta$$

$$\frac{\partial \dot{x}}{\partial \delta} = -\dot{z} \cos \alpha$$

$$\frac{\partial \dot{y}}{\partial \delta} = -\dot{z} \sin \alpha$$

$$\frac{\partial \dot{z}}{\partial \delta} = v\,(\cos \beta \cos \delta - \cos A \sin \beta \sin \delta)$$

$\beta$ (flight path angle)

$$\frac{\partial x}{\partial \beta} = \frac{\partial y}{\partial \beta} = \frac{\partial z}{\partial \beta} = 0$$

$$\frac{\partial \dot{x}}{\partial \beta} = -v\left[(\sin \beta \cos \delta + \cos A \cos \beta \sin \delta) \cos \alpha + \sin A \cos \beta \sin \alpha\right]$$

$$\frac{\partial \dot{y}}{\partial \beta} = -v\left[(\sin \beta \cos \delta + \cos A \cos \beta \sin \delta) \sin \alpha - \sin A \cos \beta \cos \alpha\right]$$

$$\frac{\partial \dot{z}}{\partial \beta} = v\,(\cos A \cos \beta \cos \delta - \sin \beta \sin \delta)$$

A (azimuth)

$$\frac{\partial x}{\partial A} = \frac{\partial y}{\partial A} = \frac{\partial z}{\partial A} = 0$$

$$\frac{\partial \dot{x}}{\partial A} = v(\sin A \sin \delta \cos \alpha - \cos A \sin \alpha) \sin \beta$$

$$\frac{\partial \dot{y}}{\partial A} = v(\sin A \sin \delta \sin \alpha + \cos A \cos \alpha) \sin \beta$$

$$\frac{\partial \dot{z}}{\partial A} = -v(\sin A \cos \delta \sin \beta)$$

r (magnitude of radial vector)

$$\frac{\partial x}{\partial r} = \frac{x}{r}$$

$$\frac{\partial y}{\partial r} = \frac{y}{r}$$

$$\frac{\partial z}{\partial r} = \frac{z}{r}$$

$$\frac{\partial \dot{x}}{\partial r} = \frac{\partial \dot{y}}{\partial r} = \frac{\partial \dot{z}}{\partial r} = 0$$

v (velocity)

$$\frac{\partial x}{\partial v} = \frac{\partial y}{\partial v} = \frac{\partial z}{\partial v} = 0$$

$$\frac{\partial \dot{x}}{\partial v} = \frac{\dot{x}}{v}$$

$$\frac{\partial \dot{y}}{\partial v} = \frac{\dot{y}}{v}$$

$$\frac{\partial \dot{z}}{\partial v} = \frac{\dot{z}}{v}$$

The equations for the partial derivatives of position and velocity components, with respect to elliptic elements, are used to compute initial conditions at time $t_o$ of the variational equations for the parameters of this type. They may also be used to estimate analytically the trajectory partial derivatives. These equations are as follows.

a (semi-major axis)

$$X_a = \frac{1}{a}\left(X - \frac{3M}{2n}\dot{X}\right)$$

$$\dot{X}_a = \frac{1}{a}\left(\dot{X} - \frac{3M}{2n}\ddot{X} - \frac{3}{2}\dot{X}\right), \qquad \ddot{X} = -\frac{\mu X}{r^3}$$

e (eccentricity)

$$X_e = -\left[a + \frac{y_\omega^2}{r(1-e^2)}\right]P + \frac{x_\omega y_\omega}{r(1-e^2)}Q$$

$$\dot{X}_e = -\frac{1}{r(1-e^2)^{1/2}}\left[\frac{y_\omega}{(1-e^2)^{1/2}}\dot{y}_\omega + n\left(\frac{a}{r}\right)^2 x_\omega y_\omega\right]P$$

$$+\frac{1}{r(1-e^2)^{1/2}}\left[\frac{y_\omega}{(1-e^2)^{1/2}}\dot{x}_\omega + n\left(\frac{a}{r}\right)^2 x_\omega^2\right]Q$$

i (inclination)

$$X_i = \frac{z}{(P_z^2 + Q_z^2)^{1/2}}W \qquad \text{where } W = P \times Q$$

$$\dot{X}_i = \frac{\dot{z}}{(P_z^2 + Q_z^2)^{1/2}}W$$

$\Omega$ (longitude of ascending node)

$$\frac{\partial x}{\partial \Omega} = -y \qquad \frac{\partial \dot{x}}{\partial \Omega} = -\dot{y}$$

$$\frac{\partial y}{\partial \Omega} = x \qquad \frac{\partial \dot{y}}{\partial \Omega} = \dot{x}$$

$$\frac{\partial z}{\partial \Omega} = 0 \qquad \frac{\partial \dot{z}}{\partial \Omega} = 0$$

$\omega$ (argument of perigee)

$$X_\omega = -y_\omega P + x_\omega Q$$

$$\dot{X}_\omega = -\dot{y}_\omega P + \dot{x}_\omega Q$$

$\tau$ (time of perigee passage)

$$X_\tau = -\dot{X}$$

$$\dot{X}_\tau = \frac{\mu X}{r^3}$$

The variational equation for initial time is like Eq. (54) with the initial conditions $X_{t_c}(t_o) = -\dot{X}(t_o)$ and $\dot{X}_{t_o}(t_o) = -\ddot{X}(t_o)$.

3.6.2.3 Differential Equation Parameter Non-homogeneous Terms

The non-homogeneous terms $\frac{\partial F}{\partial \beta}$ for the differential equation parameter variational equations are:

$\frac{C_D A}{W}$ (drag coefficient)

$$\frac{\partial F}{\partial \left(\frac{C_D A}{W}\right)} = F_3 \left(\frac{C_D A}{W}\right)^{-1}$$

$\mu$ (gravitational constant)

$$\frac{\partial F}{\partial \mu} = \frac{F_1 + F_2}{\mu} - \frac{X}{r^3}$$

$J_i$, $J_{ik}$, $\lambda_{ik}$ (oblateness parameters)

Denote the perturbative force components in the Up, East, North system (see paragraph 3.6.1) as follows:

$$g_U = -\frac{\mu}{r^2}\left[\sum_{n=2}^{5} A_n + \sum_{n=2}^{4} \sum_{m=1}^{n} B_{nm} \cos m\,(\lambda - \lambda_{nm})\right]$$

$$g_E = -\frac{\mu}{r^2}\left[\sum_{n=2}^{4} \sum_{m=1}^{n} C_{nm} \sin m\,(\lambda - \lambda_{nm})\right]$$

$$g_N = -\frac{\mu}{r^2}\left[\sum_{n=2}^{5} D_n + \sum_{n=2}^{4} \sum_{m=1}^{n} E_{nm} \cos m\,(\lambda - \lambda_{nm})\right]$$

then

$$\frac{\partial g_U}{\partial J_i} = \frac{-\mu}{r^2}\,\frac{A_i}{J_i}, \quad \frac{\partial g_E}{\partial J_i} = 0, \quad \frac{\partial g_N}{\partial J_i} = \frac{-\mu}{r^2}\,\frac{D_i}{J_i} \tag{60}$$

$$\frac{\partial g_U}{\partial J_{ik}} = \frac{-\mu}{r^2}\,\frac{B_{ik} \cos k\,(\lambda - \lambda_{ik})}{J_{ik}}, \quad \frac{\partial g_E}{\partial J_{ik}} = \frac{-\mu}{r^2}\,\frac{C_{ik} \sin k\,(\lambda - \lambda_{ik})}{J_{ik}} \tag{61}$$

$$\frac{\partial g_N}{\partial J_{ik}} = \frac{-\mu}{r^2}\,\frac{E_{ik} \cos k\,(\lambda - \lambda_{ik})}{J_{ik}} \tag{62}$$

$$\frac{\partial g_U}{\partial \lambda_{ik}} = \frac{-\mu}{r^2} kB_{ik} \sin k\,(\lambda-\lambda_{ik}), \quad \frac{\partial g_E}{\partial \lambda_{ik}} = \frac{+\mu}{r^2} kC_{ik} \cos k\,(\lambda-\lambda_{ik}) \tag{63}$$

$$\frac{\partial g_N}{\partial \lambda_{ik}} = \frac{-\mu}{r^2} kE_{ik} \sin k\,(\lambda-\lambda_{ik}) \tag{64}$$

The component terms are then rotated to the ECI system by the matrix given in paragraph 3.6.1.

### 3.6.3 Integration Methods

For the numerical integration of the differential equations described in paragraphs 3.6.1 and 3.6.2, a choice of methods is offered. They are widely known as the Adams-Moulton and the Gauss-Jackson methods, and the subroutine names are AMRK and DE6F, respectively. Both are variable-step predictor-corrector methods with automatic local truncation error control and double-precision accumulation features. Both use the Runge-Kutta methods to obtain starting values. (See Reference 13.)

The Gauss-Jackson method, utilizing $6^{th}$ differences, is of higher order and has proved to be remarkably effective in the integration of most satellite trajectories. In some restricted but well-controlled tests, this method, applied to the equations of motion, produced results that compared favorably in both speed and accuracy with more sophisticated special perturbation methods. (See Reference 14.) Because of its procedure for changing the step size, the subroutine's efficiency will drop and lose accuracy when the step size changes are extreme, as in highly eccentric orbits or upon entry into the atmosphere. In these cases the use of AMRK, in which the variable step is more stably handled, is recommended.

### 3.6.4 Interpolation

Each time the position and velocity (and their partial derivatives) of a vehicle are required, the desired quantities $X$, $\dot{X}$, $X_{p_i}$ and $\dot{X}_{p_i}$ are obtained by interpolation from the results of the integration. This technique permits an uninterrupted numerical integration, is comparatively rapid, and, as used here, is quite accurate. In particular, function values and their first and second derivatives at the two adjacent integration steps are retained to permit $5^{th}$ and $3^{rd}$ degree interpolations for position and velocity, respectively.

The method used is Hermite interpolation (Reference 15).

3.6.5 Trajectory Output

The position and velocity vectors, X and $\dot{X}$, of the vehicle are the basis of the trajectory output. From these quantities, obtained by interpolation from the results of the numerical intergration, are computed the spherical coordinates $\alpha$, $\delta$, $\beta$, A, r, v of the vehicle (see paragraph 3.2.2) and also

$$\text{geodetic latitude, } \Phi^* = \tan^{-1}\left[\frac{z}{(x^2 + y^2)^{1/2}(1 - \varepsilon)^2}\right]$$

$$\text{longitude, } \ell = \alpha - \alpha_g - \omega_e(t - t_g)$$

$$\text{height, } h = r - \frac{a_e(1 - \varepsilon)}{\left[1 - (2\varepsilon - \varepsilon^2)\dfrac{x^2 + y^2}{r^2}\right]^{1/2}}$$

These results are output in feet, degrees, and seconds.

The partial derivatives, $X_{p_i}$ and $\dot{X}_{p_i}$ ($i = 1, \ldots, n$), of vehicle position and velocity with respect to the n trajectory parameters can also be printed.

Optionally, the elements of the osculating ellipse are output. Included are the elements a, e, i, $\Omega$, $\omega$, $\tau$ (see paragraph 3.2.4), and also

$$\text{Mean anomaly, } M = E - e \sin E, \text{ where } E = \cos^{-1}\left(\frac{1 - \frac{r}{a}}{e}\right) \quad \text{(deg)}$$

True anomaly, $f = 2 \tan^{-1} \left[ \left( \frac{1+e}{1-e} \right)^{1/2} \tan \frac{E}{2} \right]$ (deg)

$$\dot{\Omega} = - \frac{3 J_2 a_e^2 \sqrt{\mu}}{2 a^{3/2} p^2} \cos i \qquad \text{(deg/day)}$$

$$\dot{\omega} = \frac{3 J_2 a_e^2 \sqrt{\mu}}{a^{3/2} p^2} \left(1 - \frac{5}{4} \sin^2 i\right) \quad \text{where } p = \frac{r^2 v^2 \sin^2 \beta}{\mu} \qquad \text{(deg/day)}$$

Apogee, $r_a = a(1 + e)$ (n mi)

Perigee, $r_p = a\,(1 - e)$ (n mi)

Keplerian Period, $P_K = \frac{2\pi a^{3/2}}{\sqrt{\mu}}$ (min)

Anomalistic Period, $P_A = P_K \left[ 1 - \frac{\frac{3}{2} J_2 a_e^2}{a^2} \left(\frac{a}{r}\right)^3 \left(1 - 3 \sin^2 \delta\right) \right]$ (min)

where $\delta$ = declination

Nodal Period, $P_N = P_A - P_K \left[ \frac{3 J_2 a_e^2 \left(1 - 5/4 \sin^2 i\right)}{\sqrt{p}\; a^2 \left(1 + e \cos \omega\right)^2} \right]$ (min)

(Formulae from Reference 16.)

### 3.6.6 Analytic Trajectory

On option, an analytic orbit can be obtained instead of an integrated orbit. The analytic trajectory consists of a Keplerian orbit with nodal regression and element decay due to atmospheric drag. The changes in elements are calculated at perigee on every $n^{th}$ revolution where n is specified These formulae are taken from Reference 17.

3.6.6.1 First Order Nodal Regression

For one revolution, for small e:

$$\Delta\Omega = \frac{-3\pi J_2 a_e^2 \cos i}{a^2 (1 - e^2)^2}$$

$$\Delta\omega = \frac{3\pi J_2 a_e^2 (4 - 5 \sin^2 i)}{2a^2 (1 - e^2)^2}$$

or

$$\frac{\Delta\Omega}{\Delta t} = \frac{-3J_2 a_e^2 \mu^{1/2}}{2a^{7/2}(1 - e^2)^2} \cos i$$

$$\frac{\Delta\omega}{\Delta t} = \frac{3J_2 a_e^2 \mu^{1/2}}{4a^{7/2}(1 - e^2)^2} (4 - 5 \sin^2 i)$$

3.6.6.2 Element Decay Due to a Rotating Atmosphere

Define scale height $H = \frac{2}{15} h + 12$ where h is the altitude in nautical miles. If $\frac{ae}{H} > 2$, the following formulae are used:

$$\Delta a = - Q \left[ 1 + \frac{1 - 8e + 3e^2}{8c (1 - e^2)} \right]$$

$$\Delta e = - Q \left( \frac{1 - e}{a} \right) \left[ 1 - \frac{3 + 4e - 3e^2}{8c (1 - e^2)} \right]$$

$$\Delta i = - B(1-e)^2 \left[ \frac{1}{c} \left(\frac{1+e}{1-e}\right) + \left(1 + \frac{f^*}{2c} + \frac{9e^2+6e-15}{8c(1-e)^2}\right) \cos^2 \omega \right] \sin i$$

$$\Delta\Omega = - B(1 - e)^2 \left[ 1 + \frac{f^*}{c} + \frac{9e^2 + 6e - 15}{8c\,(1 - e)^2} \right]$$

$$\Delta\omega = - \Delta\Omega \cos i$$

where:

$$Q = \frac{C_D A}{m} \rho_p a^2 f \frac{(1 + e)^2}{(1 - e^2)^{1/2}} \left(\frac{2\pi}{c}\right)^{1/2}$$

$$c = \frac{ae}{H}$$

$$f = 1 - \frac{2n_e}{n} (1 - e)\left(\frac{1 - e}{1 + e}\right)^{1/2} \cos i$$

$\frac{n_e}{n}$ = ratio of earth rotation rate to satellite mean motion

$\rho_p$ = density at perigee

$$B = \pi \frac{C_D A}{m} \frac{n_e}{n} a \rho_p f^{1/2} (2\pi c)^{-1/2}$$

$$f^* = \frac{e}{1 - e^2} \left( e + \frac{f - 1}{f^{1/2}} \right)$$

If $\frac{ae}{H} = c \leq 2$ then use:

$$\Delta a = -G \frac{(1+e)^{3/2}}{(1-e)^{1/2}} \left[(1-2e) I_o(c) + 2eI_1(c)\right]$$

$$\Delta e = -\frac{G}{a} \frac{(1+e)^{1/2}}{(1-e)^{1/2}} \left\{(1-e) I_1(c) + \frac{e}{2}\left[I_o(c) + I_2(c)\right]\right\}$$

$$\Delta i = -K \left\{\frac{1}{2}\left[I_o(c) - I_2(c)\right] + \cos^2 \omega \left[I_2(c) - 2eI_1(c)\right]\right\} \sin i$$

$$\Delta\Omega = -K \left[I_2(c) - 2eI_1(c)\right] \sin \omega \cos \omega$$

$$\Delta\omega = -\Delta\Omega \cos i$$

where

$$G = 2\pi \frac{C_D A}{m} a^2 \rho_p f (e^{-c})$$

$$K = \pi \frac{C_D A}{m} \frac{n_e}{n} a \rho_p \sqrt{f} (e^{-c})$$

$I_0$, $I_1$, $I_2$ = imaginary Bessel functions of the first kind.

### 3.6.7 Initial Condition Derivation (Gaussian Method)

On option, initial conditions may be calculated from two sets of RAE observations. The following procedure is adapted from Reference 18.

Let

$X_1$ = cartesian vector associated with first RAE observation

$X_2$ = cartesian vector associated with second RAE observation

$$W = \frac{X_1 \times X_2}{|X_1 \times X_2|} = \text{unit vector perpendicular to plane of}$$

observations

$$U_1 = \frac{X_1}{|X_1|} = \text{unit vector parallel to } X_1$$

$$V_1 = W \times U_1 = \text{unit vector perpendicular to } U_1 \text{ and } W$$

Compute

$$f = \frac{1}{2} (v_2 - v_1) = \frac{1}{2} \arccos \frac{X_1 \cdot X_2}{|X_1| \; |X_2|}$$

$$g_1 = \frac{\sqrt{2\mu} \, (t_2 - t_1)}{[2(|X_1| \; |X_2|)^{1/2} \cos f]^{3/2}}$$

$$g_2 = \frac{|X_1| + |X_2|}{2(|X_1| \; |X_2|)^{1/2} \cos f}$$

The following iteration is then used to find g

Set $g^{(o)} = f$

Calculate

$g_3 = \sin g^{(i)}$

$g_4 = \cos g^{(i)}$

$g_5 = \sin^3 g^{(i)}$

$g_6 = \sin^4 g^{(i)}$

$$g_7 = g_2 - g_4$$

$$g_8 = \sqrt{g_7}$$

$$g_9 = (g_7)^2$$

$$\Delta g = \frac{\frac{1}{g_7}\left(1 - \frac{g_1}{g_2}\right) + \frac{1}{g_5}\,(g^{(i)} - g_3 g_4)}{\frac{g_3}{g_9}\left(\frac{3g_1}{2g_7} - 1\right) - \frac{1}{g_6}\,[g_5 + 3(g^{(i)} - g_3)]}$$

then $g^{(i+1)} = g^{(i)} - \Delta g$.

Iterate until $|\Delta g| \leq \varepsilon$.

The Keplerian elements are then given by the following equations.

$$a = \frac{|X_1| + |X_2| - 2\sqrt{|X_1|\,|X_2|}\cos g \cos f}{2 \sin^2 g}$$

$$e \cos E_1 = 1 - \frac{|X_1|}{a}\,; \quad e \cos E_2 = 1 - \frac{|X_2|}{a}$$

$$e \sin E_1 = \frac{\cos 2g}{\sin 2g}\,[e \cos E_1 - (\cos 2g)(e \cos E_2)]$$

$$-(\sin 2g)(e \cos E_2)$$

$$E_1 = \tan^{-1} \frac{e \sin E_1}{e \cos E_1} \qquad 0 \leq E_1 < 2\pi$$

$$e = [(e \cos E_1)^2 + (e \sin E_1)^2]^{1/2}$$

$$T = t_1 - (E_1 - e \sin E_1)\frac{a^{3/2}}{\sqrt{\mu}} \quad \text{if } a > 0$$

$$i = \cos^{-1} W_z \qquad 0 \le i < \pi$$

$$\Omega = \tan^{-1} \frac{W_x}{-W_y} \qquad 0 \le \Omega < 2\pi$$

$$\cos v_1 = \frac{\cos E_1 - e}{1 - e \cos E_1}, \quad \sin v_1 = \frac{\sqrt{1-e^2} \sin E_1}{1 - e \cos E_1}$$

$$P = U_1 \cos v_1 - V_1 \sin v_1$$

$$Q = U_1 \sin v_1 + V_1 \cos v_1$$

$$\omega = \tan^{-1} \frac{P_z}{Q_z} \qquad 0 \le \omega < 2\pi$$

## 3.7 DIFFERENTIAL CORRECTION AND ASSOCIATED COMPUTATIONS

Basically, the problem of differential correction is the change, or correction, of a given set of parameters so as to achieve some specified result. In this case, the goal is to minimize the weighted sum of the squares of the differences between the (observed) radar data and the corresponding quantities computed from the observational model. The model, of course, includes the trajectory and radar parameters to be corrected.

### 3.7.1 Notation and Nomenclature

In general, matrices and vectors will be denoted by Roman capitals; their components by corresponding lower case letters, with subscripts where applicable.

| | |
|---|---|
| n | number of observed quantities |
| m | number of parameters |
| k | number of effective parameters (m minus the number of restraint equations) |
| $o_i$ | $i^{th}$ observation (may be range, azimuth, elevation, range rate, $\dot{P}$, $\dot{Q}$, P, Q) |
| $\sigma_{sj}$ | radar sigma (multiplicative weighting factor) to be applied to data type j from station s |
| $\beta_{sj}$ | radar bias (additive weighting factor) to be applied to data type j from station s |
| $O_{mc}$ | vector of weighted residuals (differences between observed and computed radar quantities) |
| P | vector of parameters |
| $\Delta P$ | correction vector for P |

| | |
|---|---|
| G | vector of bounds on solution $\Delta \mathbf{P}$ |
| A | matrix of weighted partial derivatives; $a_{ij} = \frac{\partial o_i}{\partial p_j} / \sigma$ <br> $i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m$ <br> $o_i$ determines the appropriate $\sigma$ to be applied |
| $A^T$ | A transpose |
| B | constraint matrix |

### 3.7.2 Sigmas and Biases

Usually, some empirical evidence is available as to the relative accuracy of various types of data from different stations. For this reason, two types of factors, each correlated with a particular station and data type, are used.

The most common of these are the radar sigmas. The residuals and partial derivatives of a given type of observation from a specific station are divided by the corresponding $\sigma_{sj}$. In this way, it is possible to insure that the more accurate data have a large part in determining the optimum set of parameters.

If it is known (or suspected) that a station has a constant bias in reporting either data or time, the $\beta_{sj}$ are used. These are applied to the appropriate components of $O_{mc}$ (before the division by $\sigma_{sj}$). Biases may be included with the parameters to be solved for, but only biases on basic data types.

In the succeeding paragraphs, the sigmas and biases in A and $O_{mc}$ are included implicitly.

### 3.7.3 The Unconstrained Normal

In its simplest form, differential correction involves the solution of the linearized problem $(A^T A)\ \Delta P = A^T O_{mc}$. $A^T A$ is the normal matrix. If $a_i$ is a row of A and if $A^T A \equiv 0$ initially, the normal is formed in TRACE by accumulating $A^T A = A^T A + a_i^T a_i$, $i = 1, 2, \ldots, n$, at each observation time.

### 3.7.4 The Constrained Normal

It is often desirable to impose linear constraints of the form $\Delta P = B(\Delta P') + C$ on the solution. P' is some subset of P, and C is a vector of constants. For instance, suppose the parameters to be solved for are

$p_1 = S_1$, latitude

$p_2 = S_1$, longitude

$p_3 = S_2$, latitude

$p_4 = S_2$, longitude

$p_5 = S_2$, range bias

where $S_1$, $S_2$ are two radar stations.

Then the requirement that the positions of the station relative to each other remain fixed is equivalent to the matrix equation

$$\underbrace{\begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \Delta p_3 \\ \Delta p_4 \\ \Delta p_5 \end{bmatrix}}_{\Delta P} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \Delta p_5 \\ \\ \\ \end{bmatrix}}_{\Delta P'} + \underbrace{\begin{bmatrix} \\ \\ 0 \\ \\ \\ \end{bmatrix}}_{C} \qquad (65)$$

The effective problem now is to solve the reduced system $(AB)^T(AB)(\Delta P') = (AB)^T O_{mc}$. If $a_i$ is a row of A, and if $A^TA \equiv 0$ initially, the restricted normal is formed by accumulating $A^TA = A^TA + (a_iB)^T(a_iB)$.

### 3.7.5 Bounds

Given a set of bounds $g_i$, the corrections $\Delta p_i$ to the components of P are:

(a) less (in absolute value) than $g_i$ if $g_i > 0$

(b) zero if $g_i = 0$

(c) unrestricted if $g_i < 0$

for $i = 1, 2, \ldots, m$.

If constraints are to be applied, the bounds are adjusted to the new problem.

Let $h_j = \sum_{i=1}^{m} \frac{(\text{Sign } g_i)}{(g_i)^2} b_{ij}$ $\qquad j = 1, 2, \ldots, k$

Then there will be k new bounds, $g_j'$, where

$$g_j' = \frac{1}{\sqrt{h_j}} \quad \text{if } h_j > 0$$

$$g_j' = 0 \text{ if } h_j = 0$$

$$g_j' < 0 \text{ if } h_j < 0$$

Notice that $g_j' = g_i$ for a variable not appearing in any restraint equation. Also, specifying bounds that are equal in magnitude but opposite in sign for two parameters to be corrected by equal increments will result in a zero correction to both.

3.7.6 Solution of the Normal Equations

For the purpose of this section, assume that there are m parameters $p_1, p_2, \ldots, p_m$ to be corrected. (The succeeding paragraphs require no change other than a substitution of k for m in the constrained case.)

At this point then, we have the m×m matrix $A^TA$, the vector $A^TO_{mc}$, and a set of bounds, $g_i$. The problem then is to minimize $||A\Delta P - O_{mc}||^2$

under the side condition that $\Sigma \left( \frac{\Delta p_i}{g_i} \right)^2 \leq 1$, the sum being taken over all i for which $g_i > 0$.

We can, without loss of generality, assume that $g_i \neq 0$. This is so because $g_i = 0$ implies that $\Delta p_i = 0$, and the $i^{th}$ row and column of the normal matrix may be ignored. This simply reduces the dimension of the problem.

Now, let G be the diagonal matrix so that $G_{ii} = \frac{1}{g_i}$ if $g_i > 0$; $g_{ii} = 0$ if $g_i \leq 0$. We wish to find a value of z so that the solution $\Delta P'(z)$ of the linear system $(A^T A + zG^2) \Delta P = A^T O_{mc}$ satisfies the given side condition. This involves two procedures: the choice of the best value for z, and the actual solution of the system.

3.7.6.1 Determination of z

As a start, find $\Delta P'(0)$, (the solution to $(A^T A) \Delta P = A^T O_{mc}$). If $\Sigma \left( \frac{\Delta p_i}{g_i} \right)^2 \leq 1 + \epsilon_1$, the problem is solved. ($\epsilon_1$ and $\epsilon_2$ here are suitably small positive constants.) If not, define $y(z) = \Sigma \left( \frac{\Delta p_i(z)}{g_i} \right)^2 - 1$. Now $y(0) > \epsilon_1$. Compute $y(h)$, $y(10h)$, $y(100h)$, ...; h some present constant, until either

(1) a value of $z(=kh)$ is found so that $-\epsilon_2 \leq y(z) \leq \epsilon_1$, in which case $\Delta P(z)$ is the solution, or

(2) two values of z are found so that $y(z_1) > \epsilon_1$ and $y(z_2) < -\epsilon_2$. The required value of z is now bracketed.

If (2) is the case, the next step is to choose a value $z_3$ between $z_1$ and $z_2$. $z_3 = 0.8z_1 + 0.2z_2$ where the 0.8, 0.2, are fairly arbitrary and $z_1$ and $z_2$ may have been interchanged, so that $z_3$ is closest to the value of z giving the smallest $y(z)$.

If $-\epsilon_2 \leq y(z_3) \leq \epsilon_1$, $\Delta P'(z_3)$ is the solution. Otherwise, inverse quadratic interpolation is used to obtain a new guess $z_4$. Again, if $-\epsilon_2 \leq y(z_4) \leq \epsilon_1$, $\Delta P'(z_4)$ is the solution. If not, the two values of z, from the set ($z_1$, $z_2$,

$z_3$, $z_4$), which bracket the solution most tightly, are chosen and the process is repeated from (2).

If more than twenty solutions of the linear system are required, the process is abandoned.

### 3.7.6.2 Solution of the Linear System

This section describes the procedure used in solving the linear system $(A^TA + zG^2)\,\Delta P = A^TO_{mc}$. For a discussion of the theory involved, see Section 2.

Let $C = A^TA + zG^2$. It is desired to find a matrix S so that $SCS^T = D$; S lower triangular with (-1) on the diagonal; $D = \mathrm{diag}\,(d_1, \ldots, d_n)$.

If C is a one-by-one matrix, S = -1, D = C. Now augment C by another row and column:

$$C' = \begin{pmatrix} C & d \\ d^T & \alpha \end{pmatrix}$$

Since S' must be lower triangular, with (-1) on the diagonal, and of the same order as C', it must be of the form

$$S' = \begin{pmatrix} S & 0 \\ W^T & -1 \end{pmatrix}$$

and the requirement $S'C'S'^T = D'$ is equivalent to solving for a vector W and a scalar b such that

$$\begin{pmatrix} S & 0 \\ W^T & -1 \end{pmatrix} \begin{pmatrix} C & d \\ d^T & \alpha \end{pmatrix} \begin{pmatrix} S^T & W \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} D & 0 \\ 0 & b \end{pmatrix} = D' \qquad (66)$$

It is easily verified that

$$W = S^T D^{-1} S\, d \quad \text{and}$$

$$b = \alpha - W^T d$$

satisfy the requirements.

The computations follow the above outline, starting with the two-by-two matrix $C_{ij}$, i = 1, 2,; j = 1, 2 and continuing until the decomposition

$$\begin{pmatrix} S & 0 \\ W^T & -1 \end{pmatrix} \begin{pmatrix} A^T A + zG^2 & A^T O_{mc} \\ O_{mc}^T A & O_{mc}^T O_{mc} \end{pmatrix} \begin{pmatrix} S^T & W \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} D & 0 \\ 0 & \alpha \end{pmatrix} \qquad (67)$$

has been found.

Carrying out the multiplication indicated on the left side of the equation shows that the m-dimensional vector W is the solution to the linear system.

3.7.6.3 Residuals Prediction

$\|O_{mc}^P\| = \|A\Delta P - O_{mc}\|$ is computed from the augmented normal matrix:

$$\|A\Delta P - O_{mc}\|^2 = [(A^T A\ \Delta P) \cdot \Delta P] - 2\,[(A^T O_{mc}) \cdot \Delta P] + O_{mc}^T O_{mc} \qquad (68)$$

3.7.6.4 The Inverse Normal

$$(A^T A)^{-1} = S^T D^{-1} S \qquad (69)$$

with S and D as defined in paragraph 3.7.6.2.

3.7.7 Convergence of the Differential Correction Process

The $\|O_{mc}\|$ is a measure of how well the orbit, computed on the basis of a given set of parameters P, fits the observed data. $\|O_{mc}^P\|$, computed as in paragraph 3.7.6.3, is an approximation to $\|O_{mc}\|$, which would be obtained by replacing P by P + $\Delta$P. This approximation would be exact if the least squares problem were linear; that is, if P were in a sufficiently small neighborhood of the minimum point.

Convergence, then, is defined as being that point at which further corrections corrections to P would produce no significant decrease in $\|O_{mc}\|$; i.e., no over-all improvement of the residuals. The criteria used are

$\dfrac{\|O_{mc}\| - \|O^P_{mc}\|}{\|O_{mc}\|} \leq \epsilon_1$ or $\|O_{mc}\| \cdot n^{-1/2} \leq \epsilon_2$ where n is the number of observations and $\epsilon_1$ and $\epsilon_2$ are input quantities.

If $\|O_{mc}\|$ is decreasing with each iteration, the process is converging and the bounds are expanded at each step (by a multiplicative factor $\beta_1$) to permit faster convergence. On the other hand, if $\|O_{mc}\|$ is increasing from one iteration to the next, the process is diverging and the last corrections are presumed to have altered P too drastically. In this situation the previous values of P and the corresponding normal matrix are retrieved and resolved with tighter bounds. The new bounds $g_i'$ are such that the weighted length $\|G' \cdot \Delta P'\|$ of the solution is reduced to $\beta_2$ times its previous value.

3.7.8 The Correlation Matrix

If the mathematical model is exact, if the observations are linear functions of the parameters, if the observation errors have mean zero and are independent, and if the input values of $\sigma_{sj}$ are correct, then the inverse of the normal matrix is the variance-covariance matrix of the parameters, due to the random errors in the observations. (For a proof of this, see Section 2.) If the elements of this matrix are given as $c_{ij}$, the corresponding correlation matrix has elements

$$c'_{ij} = \frac{c_{ij}}{\sqrt{c_{ii}\, c_{jj}}} \qquad i, j, = 1, 2, \ldots, m. \tag{70}$$

If all of the above assumptions are true except that all of the $\sigma_{sj}^2$ values are in error by a constant multiplicative factor, then the values in the variance-covariance matrix will also all be in error by the same factor.

## 3.8 ERROR ANALYSIS

It is often desirable to analyze the basic statistics involved in a particular orbit determination problem. This essentially entails a determination of the effects that specified sources of error have on the precision of the least squares parameters. These sources of error, for example, may be inaccuracies in station locations, random errors in observations, or errors in differential equation parameters.

The error analysis procedure does not require an actual determination of the orbit, nor does it require actual observations; however, the data types and data intervals must be specified for each station. A basic error analysis may then be obtained by simple matrix manipulation.

### 3.8.1 Notation and Nomenclature

- $n$ number of "observed" quantities
- $m$ number of least squares parameters
- $k$ number of parameters (other than least squares parameters), which are considered in error
- $\sigma_{sj}$ radar sigma to be applied to data type j from station s
- $W^{1/2}$ diagonal matrix of order n with $\sigma_{sj}^{-1}$ for each "observation" as elements
- $P$ vector of least squares parameters, or "P-parameters"
- $Q$ vector of parameters ("Q-parameters") considered in error (other than least squares parameters)
- $C(Q)$ variance-covariance matrix of the specified Q-parameters
- $A_p$ $n \times m$ matrix of weighted partial derivatives of the observations with respect to the least squares parameters;
  $A_p = W^{1/2} \frac{\partial O_c}{\partial P}$
- $A_q$ $n \times k$ matrix of weighted partial derivatives of observations with respect to Q-parameters

$$A_q = W^{1/2} \frac{\partial O_c}{\partial Q}$$

$X_t$ $(x, y, z, \dot{x}, \dot{y}, \dot{z})^T$, position and velocity vector at time t

$R_t$ $(\alpha, \delta, \beta, A, r, v)^T$, spherical coordinates at time t.

$\Xi_t$ $(\xi, \eta, \zeta, \dot{\xi}, \dot{\eta}, \dot{\zeta})^T$, orbit plane coordinates; $\xi$-radial $\eta$-in track, $\zeta$-cross track

$E_t$ $(a, e, i, \Omega, \omega, \tau)^T$, elements at time t

$T_t$ (period, apogee, perigee)$^T$ at time t.

### 3.8.2 The Normal Matrix

The normal matrix is formed in TRACE by accumulating $A^TA = \sum_{i=1}^{n} a_i^T a_i$ where $a_i$ is the $i^{th}$ row of the matrix $A = (A_p, A_q)$. In terms of P and O parameters the $A^TA$ matrix is:

$$A^TA = \left(\begin{array}{c|c} A_p^TA_p & A_p^TA_q \\ \hline A_q^TA_p & A_q^TA_q \end{array}\right) \tag{71}$$

This accumulation is made in double precision in error analysis applications only.

### 3.8.3 $(A_p^TA_p)^{-1}$

The matrix is the variance-covariance matrix of the least squares (P) parameters due to random errors in the observations. (The random errors in the observations are specified by the $\sigma_{sj}$.) In the case of the orbital parameters, the uncertainties given by this covariance matrix $C(P') = (A_p^TA_p)^{-1}$ apply only at the initial time $t_o$.

### 3.8.4 Effect of Q-parameter Errors on $(A_p^T A_p)^{-1}$

In paragraph 3.8.3, only the effect of observation errors on the set of least squares parameters was considered. However, station location errors and differential equation parameter errors will also contribute to the uncertainty of the P-parameters. By including these "Q" effects a new covariance matrix is given by:

$$C(P'') = (A_p^T A_p)^{-1} + (A_p^T A_p)^{-1} A_p^T A_q C(Q) A_q^T A_p (A_p^T A_p)^{-1}$$

$$= C(P') + \left(\frac{\partial P''}{\partial Q}\right) C(Q) \left(\frac{\partial P''}{\partial Q}\right)^T \tag{72}$$

where $\frac{\partial P''}{\partial Q} = -(A_p^T A_p)^{-1} A_p^T A_q$. Again, this variance-covariance matrix applies at the initial time $t_o$. (Note: The C(Q) matrix contains, essentially, the uncertainties in the Q-parameters and must be input.)

### 3.8.5 Transformation

Both C(P') and C(P'') are referred to the initial time. Since uncertainties in initial coordinates do not satisfactorily describe trajectory uncertainties, it may be desired to translate these covariance matrices into other coordinate systems and to times other than $t_o$. This is sometimes called the "updating" process. The general transformation equation is

$$C(X_t) = \left(\frac{\partial X}{\partial P_o}\right) C(P') \left(\frac{\partial X}{\partial P_o}\right)^T + \left(\frac{\partial X}{\partial Q_o} - \frac{\partial X}{\partial P_o} \frac{\partial P''}{\partial Q_o}\right) C(Q) \left(\frac{\partial X}{\partial Q_o} - \frac{\partial X}{\partial P_o} \frac{\partial P''}{\partial Q_o}\right)^T \tag{73}$$

Transformations to other coordinate systems are accomplished as follows. (In all cases the result is a sum of two terms: The first gives the effect of observational errors only; Q-parameter effects arise from the second. Both the first term and the sum can be printed.)

$$C(\Xi_t) = \left(\frac{\partial \Xi_t}{\partial X_t}\right) C(X_t) \left(\frac{\partial \Xi_t}{\partial X_t}\right)^T \quad \text{(orbit plane)} \tag{74}$$

$$C(R_t) = \left(\frac{\partial R_t}{\partial X_t}\right) C(X_t) \left(\frac{\partial R_t}{\partial X_t}\right)^T \quad \text{(spherical)} \tag{75}$$

$$C(E_t) = \left(\frac{\partial E_t}{\partial R_t}\right) C(R_t) \left(\frac{\partial E_t}{\partial R_t}\right)^T \quad \text{(element)} \tag{76}$$

$$C(T_t) = \left(\frac{\partial T_t}{\partial R_t}\right) C(R_t) \left(\frac{\partial T_t}{\partial R_t}\right)^T \quad \text{(period, apogee, perigee)} \tag{77}$$

### 3.8.6 Transformation Partial Derivatives

The following formulae are used for the transformation partial derivative matrices in paragraph 3.8.5.

Orbit plane coordinates: $\left(\frac{\partial \Xi_t}{\partial X_t}\right)$

$$\xi = \frac{X}{|X|} \quad \left(\text{where } X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ for this and the following equation only}\right)$$

$$\zeta = \frac{\xi \times \dot{X}}{|\xi \times \dot{X}|}$$

$$\eta = \zeta \times \xi$$

Let $\psi$ be the matrix $(\xi, \eta, \zeta)$, then

$$\frac{\partial \Xi_t}{\partial X_t} = \begin{pmatrix} \psi^T & 0 \\ 0 & \psi^T \end{pmatrix}$$

Spherical coordinates: $\left(\frac{\partial R_t}{\partial X_t}\right)$

$\alpha$ (right ascension)

$$\frac{\partial \alpha}{\partial x} = \frac{-y}{x^2 - y^2}$$

$$\frac{\partial \alpha}{\partial y} = \frac{x}{x^2 - y^2}$$

$$\frac{\partial \alpha}{\partial z} = \frac{\partial \alpha}{\partial \dot{X}} = 0$$

$\delta$ (declination)

$$\frac{\partial \delta}{\partial x} = \frac{-xz}{r^2 \sqrt{x^2 + y^2}}$$

$$\frac{\partial \delta}{\partial y} = \frac{-yz}{r^2 \sqrt{x^2 + y^2}}$$

$$\frac{\partial \delta}{\partial z} = \frac{\sqrt{x^2 + y^2}}{r^2}$$

$$\frac{\partial \delta}{\partial \dot{X}} = 0$$

$\beta$ (flight path angle)

$$\frac{\partial \beta}{\partial X} = \frac{1}{r\sqrt{v^2 - \dot{r}^2}} \left( \frac{X\dot{r}}{r} - \dot{X} \right)^T$$

$$\frac{\partial \beta}{\partial \dot{X}} = \frac{1}{r\sqrt{v^2 - \dot{r}^2}} \left( \dot{X}\,\frac{r\dot{r}}{v^2} - X \right)$$

A (azimuth)

$$\frac{\partial A}{\partial x} = \frac{\dot{y}\,(r\dot{z} - z\dot{r}) - (x\dot{y} - y\dot{x})\left(x\dot{z} - z\dot{x} + \frac{xz\dot{r}}{r}\right)\frac{1}{r}}{(v^2 - \dot{r}^2)(x^2 + y^2)}$$

$$\frac{\partial A}{\partial y} = \frac{-\dot{x}\,(r\dot{z} - z\dot{r}) + (x\dot{y} - y\dot{x})\left(y\dot{z} - z\dot{y} + \frac{yz\dot{r}}{r}\right)\frac{1}{r}}{(v^2 - \dot{r}^2)(x^2 + y^2)}$$

$$\frac{\partial A}{\partial z} = \frac{\dot{r}\,(x\dot{y} - y\dot{x})}{r^2(v^2 - \dot{r}^2)}$$

$$\frac{\partial A}{\partial \dot{x}} = -\frac{(y\dot{z} - z\dot{y})}{r(v^2 - \dot{r}^2)}$$

$$\frac{\partial A}{\partial \dot{y}} = \frac{(x\dot{z} - z\dot{x})}{r(v^2 - \dot{r}^2)}$$

$$\frac{\partial A}{\partial \dot{z}} = \frac{-r}{\dot{r}} \left( \frac{\partial A}{\partial z} \right)$$

r (radius)

$$\frac{\partial r}{\partial X} = \frac{X^T}{r}$$

$$\frac{\partial r}{\partial \dot{X}} = 0$$

v (velocity)

$$\frac{\partial v}{\partial X} = 0$$

$$\frac{\partial v}{\partial \dot{X}} = \frac{\dot{X}^T}{v}$$

Elements: $\left(\frac{\partial E_t}{\partial R_t}\right)$

a (semi-major axis)

$$\frac{\partial a}{\partial r} = \frac{2}{(2-\lambda)^2} \qquad \text{where } \lambda = \frac{rv^2}{\mu} \qquad (\lambda \neq 2)$$

$$\frac{\partial a}{\partial v} = \frac{\lambda r}{v} \left(\frac{\partial a}{\partial r}\right)$$

$$\frac{\partial a}{\partial \alpha} = \frac{\partial a}{\partial \delta} = \frac{\partial a}{\partial \beta} = \frac{\partial a}{\partial A} = 0$$

e (eccentricity)

$$\frac{\partial e}{\partial r} = \frac{p\ (\lambda-1)}{r^2 e} \qquad \text{where } p = a\ (1-e^2) \qquad (e \neq 0)$$

$$\frac{\partial e}{\partial v} = \frac{2r}{v} \left(\frac{\partial e}{\partial r}\right)$$

$$\frac{\partial e}{\partial \beta} = \frac{-y_\omega}{a} \qquad \text{where } y_\omega = \frac{r\lambda}{e} \sin \beta \cos \beta$$

$$\frac{\partial e}{\partial \alpha} = \frac{\partial e}{\partial \delta} = \frac{\partial e}{\partial B} = \frac{\partial e}{\partial A} = 0$$

i (inclination)

$$\frac{\partial i}{\partial \delta} = \sin(\alpha - \Omega)$$

$$\frac{\partial i}{\partial A} = -\cos\delta\cos(\alpha - \Omega)$$

$$\frac{\partial i}{\partial \alpha} = \frac{\partial i}{\partial \beta} = \frac{\partial i}{\partial r} = \frac{\partial i}{\partial v} = 0$$

$\Omega$ (longitude of ascending node)

$$\frac{\partial \Omega}{\partial \delta} = \frac{-\cos(\alpha - \Omega)}{\tan i} \qquad i \neq 0$$

$$\frac{\partial \Omega}{\partial A} = \frac{-\sin\delta}{\sin^2 i}$$

$$\frac{\partial \Omega}{\partial \alpha} = 1$$

$$\frac{\partial \Omega}{\partial \beta} = \frac{\partial \Omega}{\partial r} = \frac{\partial \Omega}{\partial v} = 0$$

$\omega$ (argument of perigee)

$$\frac{\partial \omega}{\partial \delta} = \frac{\cos^2(\alpha - \Omega)}{\cos A} \qquad A \neq \frac{\pi}{2}$$

$$\frac{\partial \omega}{\partial \beta} = 2 + \frac{x_\omega}{ae} \qquad \text{where } x_\omega = \frac{p-r}{e}$$

$$\frac{\partial \omega}{\partial A} = \frac{\cos\delta\sin(\alpha - \Omega)}{\sin i}$$

$$\frac{\partial \omega}{\partial r} = \frac{y_\omega}{er^2}$$

$$\frac{\partial\omega}{\partial v} = \frac{2y_{\omega}}{erv}$$

$$\frac{\partial\omega}{\partial\alpha} = 0$$

$\tau$ (time of perigee passage)

$$\frac{\partial\tau}{\partial\beta} = \frac{-\sqrt{p}\ x_{\omega}}{e\sqrt{\mu}}$$

$$\frac{\partial\tau}{\partial r} = \frac{-ay_{\omega}}{\sqrt{\mu p}} \left(\frac{p+e^2 r}{r^2 e}\right) + \frac{3}{2}\sqrt{\frac{a}{\mu}}\ M\left(\frac{\partial a}{\partial r}\right)$$ where M is mean anomaly

$$\frac{\partial\tau}{\partial v} = \frac{-2ray_{\omega}}{v\sqrt{\mu p}} \left(\frac{p+e^2 r}{r^2 e}\right) + \frac{3}{2}\sqrt{\frac{a}{\mu}}\ M\left(\frac{\partial a}{\partial v}\right)$$

$$\frac{\partial\tau}{\partial\alpha} = \frac{\partial\tau}{\partial\delta} = \frac{\partial\tau}{\partial A} = 0$$

Period, Apogee, Perigee: $\left(\frac{\partial T_t}{\partial T_t}\right)$

$$\frac{\partial T_t}{\partial R_t} = \frac{\partial T_t}{\partial E_t}\ \frac{\partial E_t}{\partial R_t}$$

p (period)

$$\frac{\partial p}{\partial a} = \frac{3}{2}\ \frac{p}{a} = 3\pi\sqrt{\frac{a}{\mu}}$$

$r_a$ (apogee)

$$\frac{\partial r_a}{\partial a} = \frac{r_a}{a} = 1+e$$

$$\frac{\partial r_a}{\partial e} = a$$

$r_p$ (perigee)

$$\frac{\partial r_p}{\partial a} = \frac{r_p}{a} = 1-e$$

$$\frac{\partial r_p}{\partial e} = -a$$

(All other terms of $\frac{\partial T_t}{\partial E_.}$ are zero.)

# SECTION 4

# PROGRAM STRUCTURE

## 4.1 GENERAL

TRACE is written in the FORTRAN language, to be used with the IBM 709/7090 FORTRAN Monitor System. The basic construction of the program is a series of independent links, which are connected by the CHAIN feature of FORTRAN. Within each link there is a series of large blocks, or major subroutines, each of which makes use of many smaller routines.

This design was chosen for TRACE for several reasons. First, the flow of computation is easy to follow and understand, both in general and in detail, by relative newcomers as well as by the authors. This is an important consideration in facilitating modifications to an intricate but continually expanding program such as TRACE. Because of the subroutine structure of TRACE, most of the presently projected additions can be made by changing one or two routines with no possibility of interfering with contiguous segments.

TRACE is restricted to use with the 709/7090 by only one characteristic—the occasional utilization of FAP. In general, the FAP subroutines are short and few. In only one link (TRAIN, the tracking data input link) do they play a major part, and even in this case they can be simply replaced. It was felt desirable to replace the FORTRAN input statements by buffering routines designed specially to handle large amounts of fixed format radar observation data cards.

TRACE is therefore an extremely flexible program partitioned into five major links. A description of these links follows and general flow charts (Figures 10 through 15) appear at the end of this section.

## 4.2 THE LINKS

### 4.2.1 CHAIN

This is the only program that must be executed regardless of the mode in which TRACE is to be used, CHAIN is divided into three sections. The first section reads basic data, but not station locations, or tracking data, or the specifications for data generation or error analysis. It also prints a header, sets several options to their nominal values, and computes the Julian Date and the orientation of the Earth. The second section initializes the trajectory parameters, and reads in extra input for GAIN or FEIGN, if the corresponding mode is being executed.

The third section is executed only during the orbit determination mode of MAIN (see paragraph 4.2.3). It contains the differential correction procedure and the corresponding output. Control is alternated between MAIN and this section of CHAIN during the orbit determination mode.

### 4.2.2 TRAIN

The tracking input link, TRAIN, reads radar station location and observation data. This data may be on the BCD input tape produced by the IBM 1401, or on a binary tape previously written by TRAIN. For real-time tracking exercises, the card reader will be used. The observations are sorted chronologically, and a compacted list is produced, which eliminates storage corresponding to blanks in the information reported. In this way, approximately two thousand observations can be handled in core (on a 32K machine) without resorting to intermediate tapes.

On option, TRAIN produces a binary tape containing the sorted and compacted observation data, to be used either on successive runs or the next case of the same run. This data has been transformed to the standard set (R, A, E, $\dot{R}$, $\dot{P}$, $\dot{Q}$, P, Q at present), and all units have been converted to an earth-radii-minutes-radians system. Observation times are reduced to minutes from midnight of epoch.

In addition, TRAIN prints the tracking input, and decodes and prints input concerning parameters to be solved for by differential correction.

### 4.2.3 MAIN

MAIN has two modes of operation: trajectory only, and orbit determination. The first is a straightforward computation of the trajectory determined by the initial conditions, using whichever of the available methods has been designated. Output is a time history of position and velocity of the vehicle, and various other related quantities, at any specified combination of print intervals.

The orbit determination mode utilizes both the trajectory block and a radar block. These two segments combine to produce residuals and partial derivatives of the observations with respect to the parameters being studied. The original nonlinear problem is solved iteratively by differential correction. Residuals are formed, corrections computed and applied and the entire process is repeated either a specified number of times or until convergence. Convergence is defined to be the point at which no further improvement can be predicted in the residuals (the differences between measured and computed observations). Divergence, defined as an over-all increase in the residuals, is also possible. In this case, the last best solution is retrieved and the corresponding system re-solved with more stringent bounds on the corrections.

Output from MAIN in this mode consists of the residual at each observation time, and the pertinent results from the differential correction routines including initial conditions, corrections, and the correlation matrix. The trajectory output and a time history of the normal matrix and its inverse are also available if desired.

### 4.2.4 GAIN

Steering data for the radar stations must be supplied in the form of separate listings for each station. Although the computations involved are basically those of the computed observations produced chronologically in MAIN, the required output form, station-by-station listings, imposes a considerable sorting problem.

Utilizing the trajectory block and a simplified radar block, GAIN proceeds so as to completely fill core with chronological ephemeris data, which is then sorted into listings that are station-by-station for the period of the data in core. (Further sorting could be added upon completion if a station-by-station listing for a longer time period is desired.)

GAIN requires as input a list of station locations and a definition of the data configuration desired from each. Any function, not necessarily an observation, could be coded and selected for output. Normally, however, output will be some type of radar observation. Data is produced only for those periods during which the vehicle is visible to the station; optionally, rise and set times only are calculated.

#### 4.2.5 FEIGN

The simulation link, FEIGN, is designed to permit studies of the large matrices involved in tracking system design without requiring their generation by one program and manipulation by another. To save storage space, the simulated data is not computed explicitly, but is instead inferred from a list of data types and frequencies for given stations. FEIGN employs the trajectory and radar blocks exactly as they are used in the differential correction path of MAIN. However, since there are no actual observations, this link checks for visibility at each point and computes derivatives, etc., only when physically applicable.

The main matrix calculation being made in FEIGN at present is the calculation and inversion of the normal matrix associated with the parameters being studied (i.e., P-parameters). The inverse of the normal matrix is the covariance matrix of the P-parameters. This covariance matrix may be updated in time (by use of the variational equation partials) and transformed to other systems. These other systems are: Cartesian; Orbit Plane; Spherical; Element; and Period, Apogee, Perigee. The effect of specified parameter errors (Q-parameters) may be included in any of the above matrices. Any combination of matrices, with or without the Q-parameter effects, may be output.

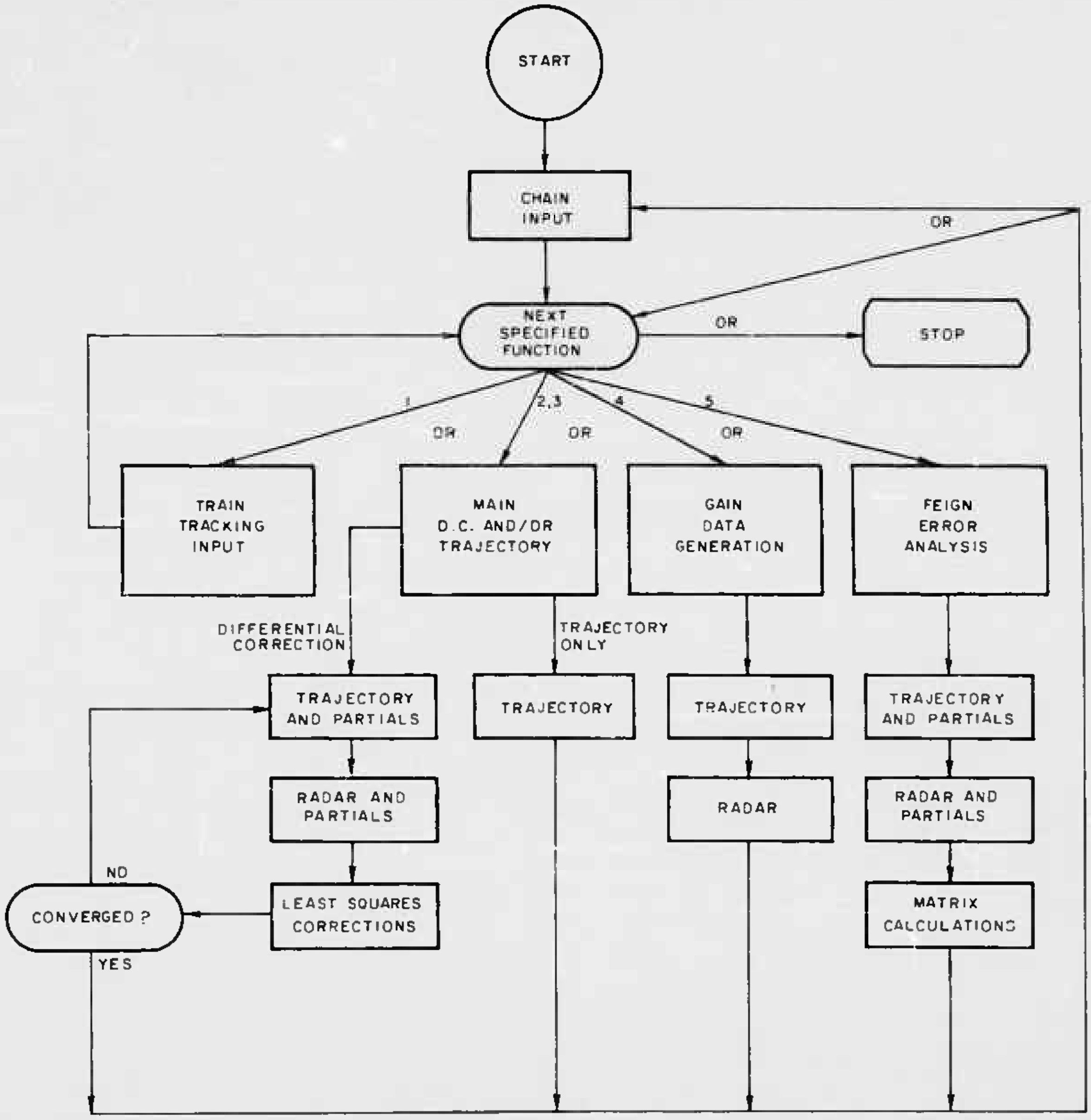



Figure 10. General Flow Chart for TRACE

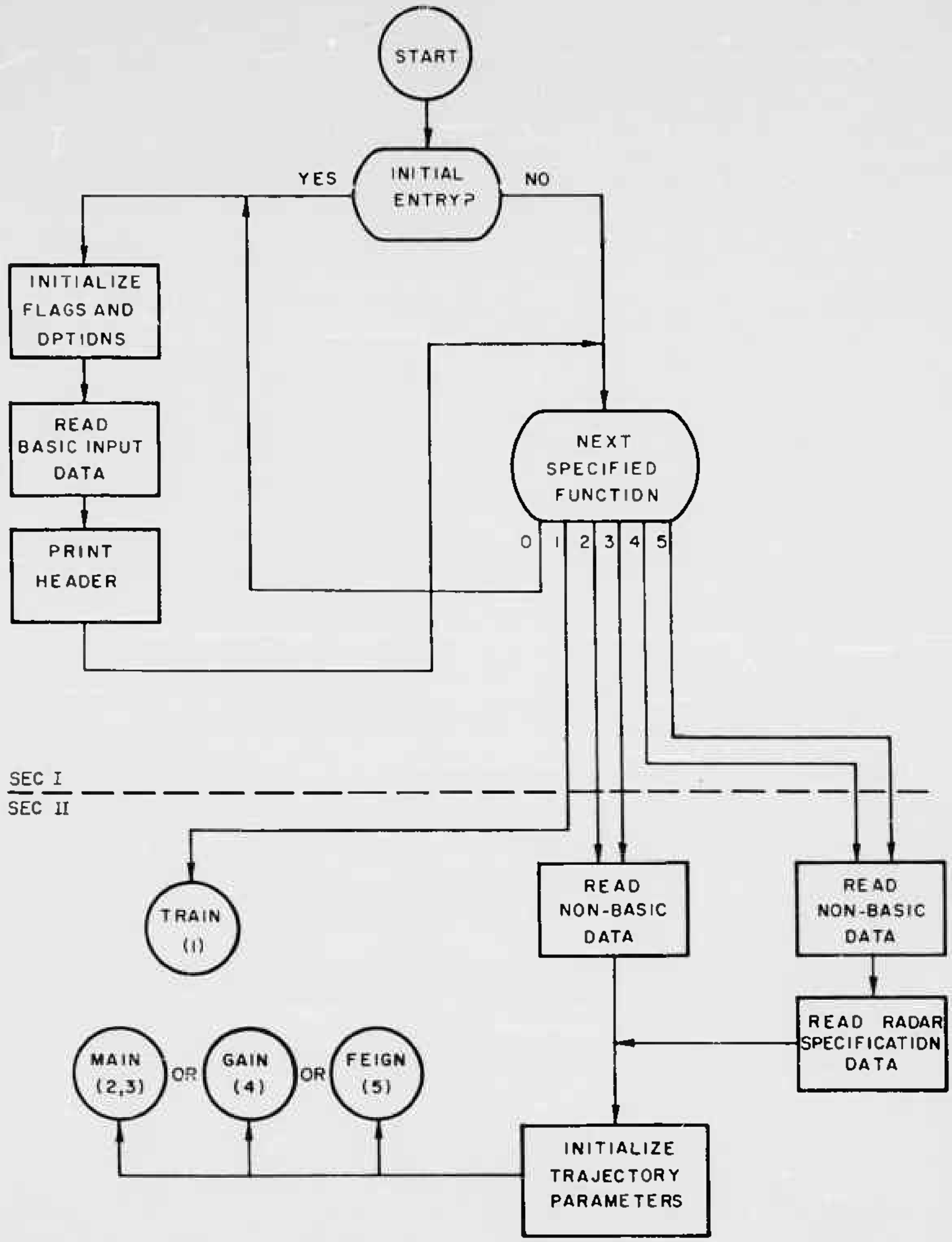



Figure 11. Flow Chart of CHAIN Link

TRAIN

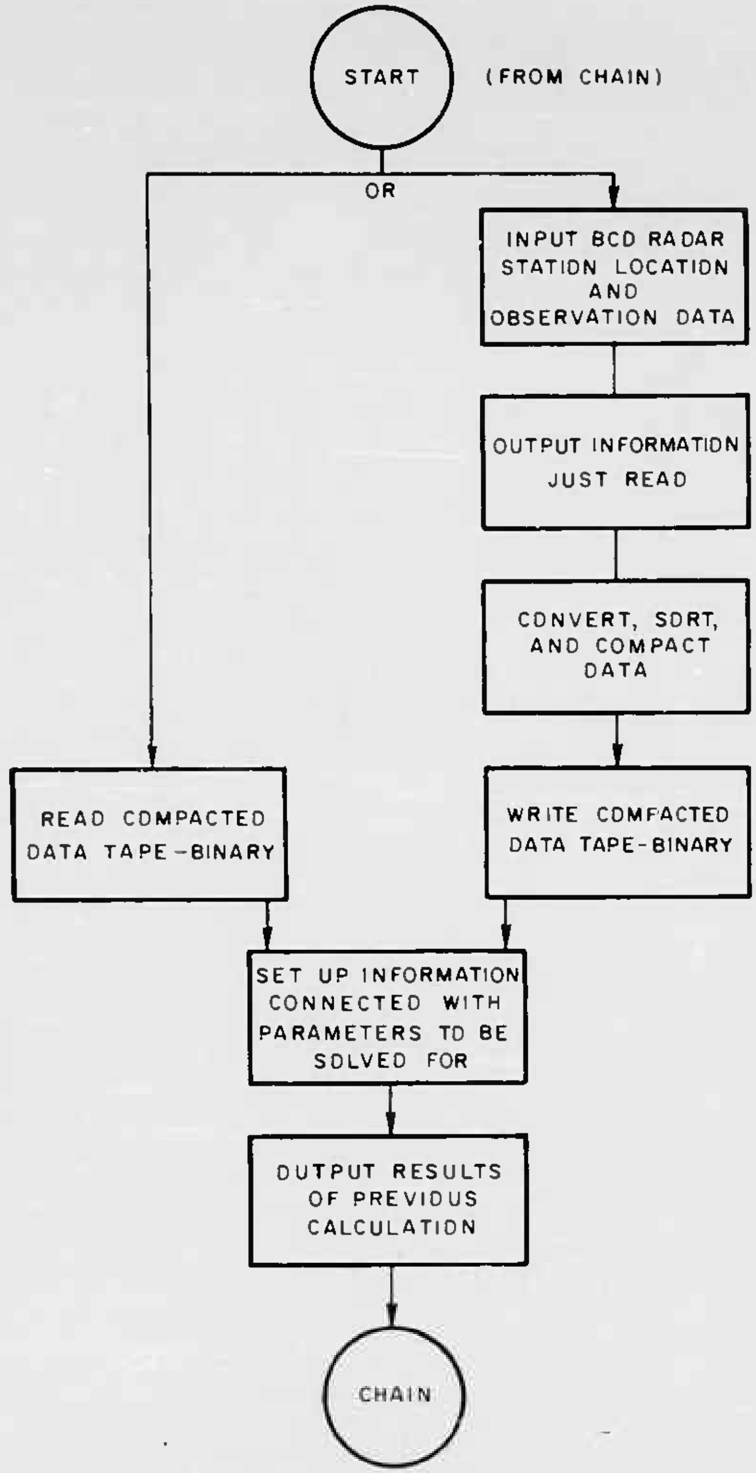



Figure 12. Flow Chart of TRAIN Link

MAIN

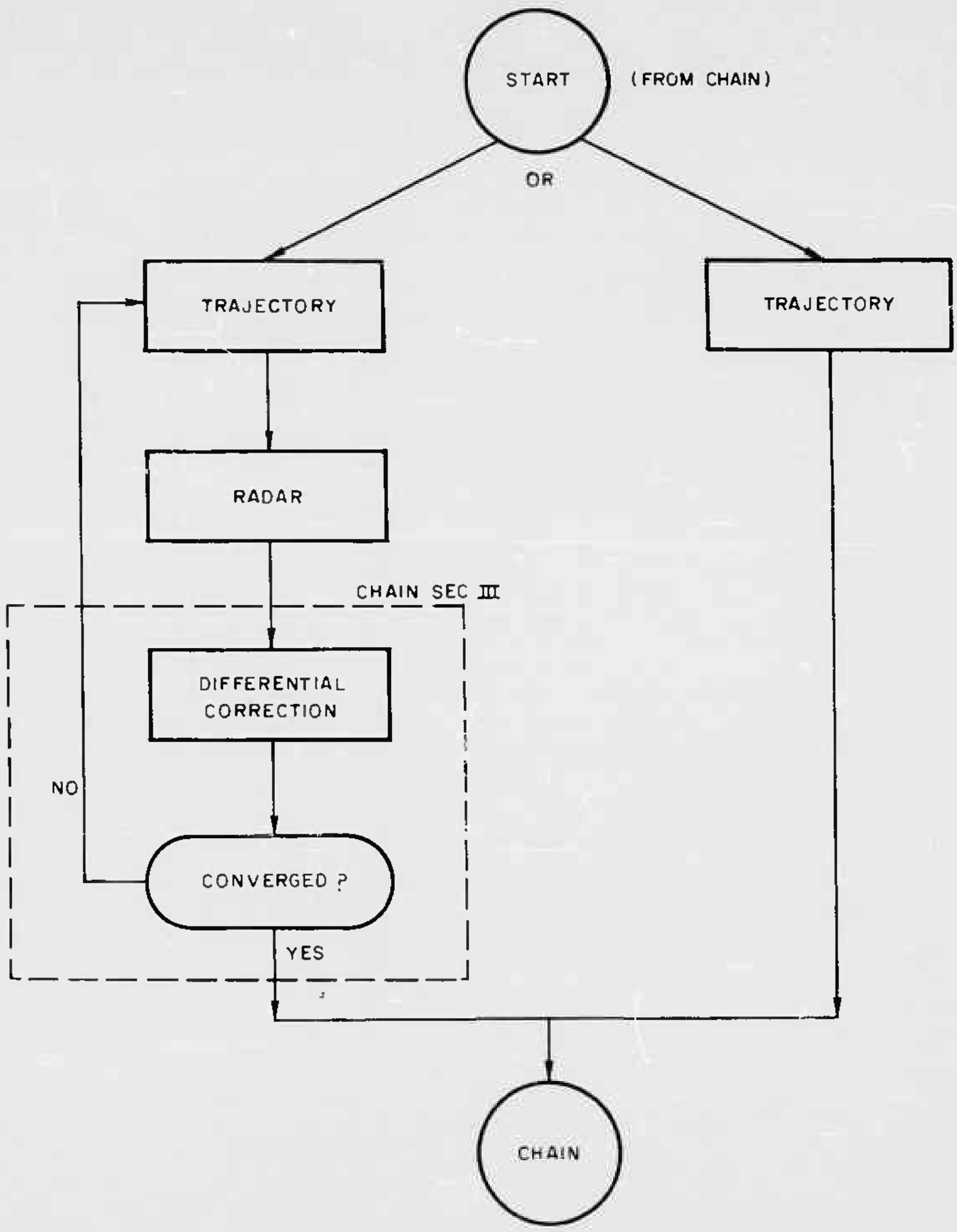



Figure 13. Flow Chart of MAIN Link

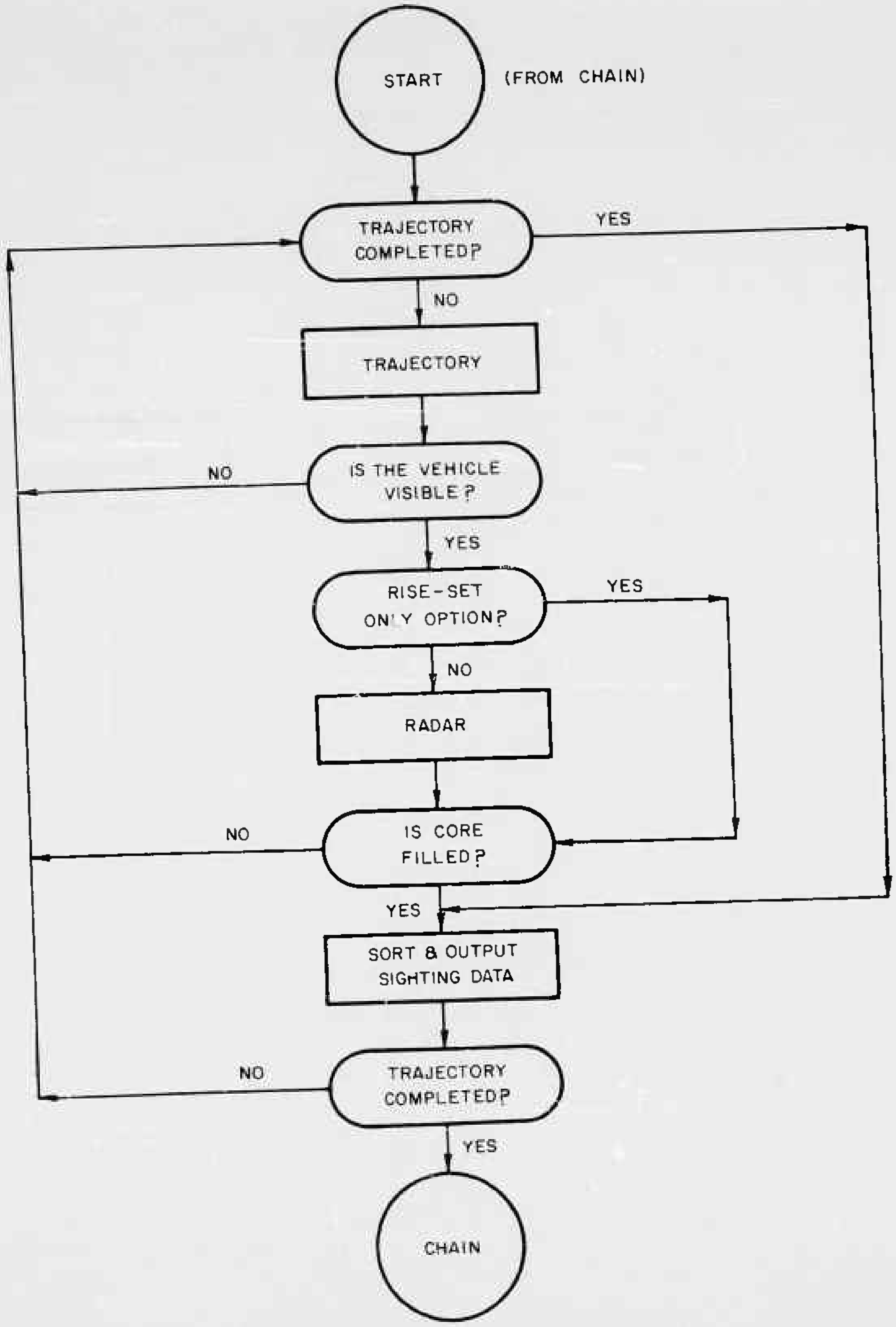



Figure 14. Flow Chart of GAIN Link

FEIGN

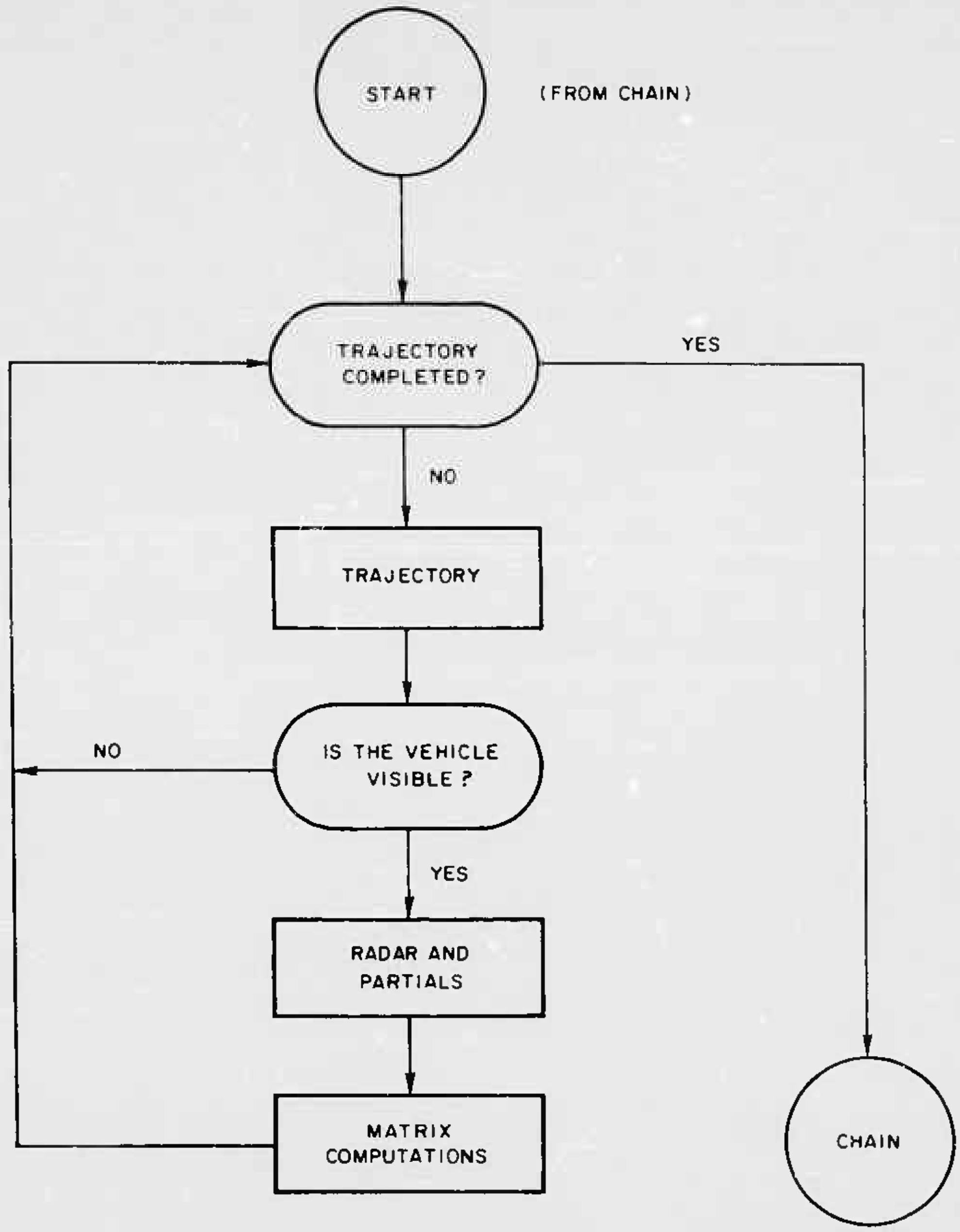



Figure 15. Flow Chart of FEIGN Link

FEIGN

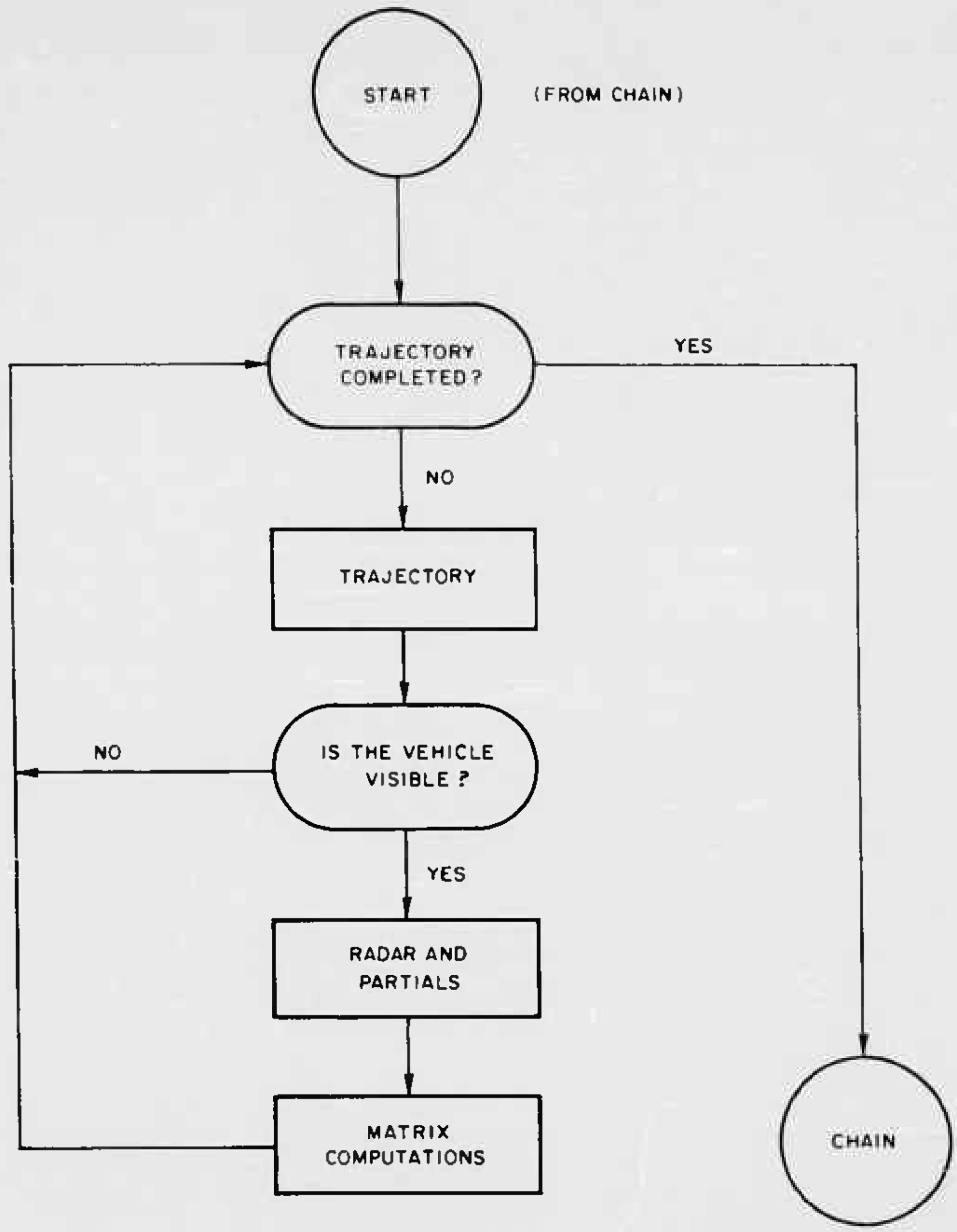



Figure 15. Flow Chart of FEIGN Link

# SECTION 5

# USAGE

## 5.1 INTRODUCTION

This section describes the input, deck setup, and output connected with the use of TRACE (in Sections 5.2, 5.2.6 and 5.3, respectively). It is designed to answer most questions pertaining to the use of the program. Each function-trajectory, tracking, data generation, and error analysis - is treated separately. A detailed explanation of each is contained in Section 1.

## 5.2 INPUT

This section is divided into six parts. The first is concerned with the Basic Data input common to all functions. The next four parts cover the functions of trajectory, tracking, data generation, and error analysis. Each part, together with the Basic Data input, is independent and completely describes the input for the function involved. Finally, the arrangement of the program and input deck is described.

TRACE utilizes the following four types of load sheets for input:

- FINP for Basic Data
- Station Location Data
- Radar Observation Data
- Error Analysis and Data Generation Specification

Sample load sheets and summary descriptions are included at the end of each part.

The following three points of information will make the FINP load sheets easier to use.

a. Although the load sheet imposes an order on the input, the actual order of the cards is (almost) immaterial, the only restriction being that all values with nonsymbolic locations are located relative to the last previous symbol. In the case of two appearances of the same location symbol, the last value read is the effective input.

b. A prefix (Columns 1, 19, 37, and 55) determines the mode of input. A blank indicates that the following value is to be read as a floating point number; an I, as a fixed point decimal; D, BCD (Hollerith); and B, as an octal. The END cards are used because the prefix E terminates the FINP read.

c. Any card for which no value appears may be omitted. Blank fields are ignored except for D prefix (BCD).

### 5.2.1 Basic Data

By definition, Basic Data is that which is common to all functions. The required Basic Data includes the list of functions to be performed, the specification of the trajectory (date, time, and initial conditions), the force model to be assumed, and the constants and parameters to be used in the trajectory integration. (The force model, constants, and parameters are "required" data, but standard values are provided. The replacement of these quantities is thus "optional." See the appendix for a list of the standard values.)

There are also options, which are common to all functions, and thus are contained in the Basic Data input. These include identification information, specification of the ballistic (drag) coefficient and atmosphere model, selection of other-body perturbations, and output print time specification.

A line-by-line explanation of the load sheet follows. An asterisk refers to a special feature, which is explained below the example.

#### 5.2.1.1 Required Input

Line 1 - Functions To Be Performed

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | D | ITIN | 1 | 2 | 4 | | |

This line contains the ordered list of all functions TRACE is expected to perform on a given run, using the code:

| | | |
|---|---|---|
| tracking data input | = | 1 |
| tracking computations | = | 2 |
| trajectory only | = | 3 |
| data generation | = | 4 |
| error analysis | = | 5 |

Up to 12 functions may be selected. When this list is exhausted, TRACE will reset certain standard options, prepare to run another sequence of functions, and read Basic Data, or stop if there is none. The example given would be for a tracking case (functions 1 and 2) followed by a data generation case.

Lines 2 through 8 - Epoch

| | | | | | |
|---|---|---|---|---|---|
| * 2 | | I | YEAR | 1963 | |
| 3 | | I | MNTH | 6 | |
| 4 | | I | DAY | 15 | |
| 5 | | | TZNE | 0 | |
| 6 | | | HR | 12 | |
| 7 | | | MIN | 45 | |
| 8 | | | SEC | 15. 5 | |

The year, month, and day should be input. The X axis is then directed to the vernal equinox (see paragraph 3. 1. 1). (* Line 2. If the year is input negative, the X axis will be directed to the longitude of Greenwich.) The hour, minute, and second refer to midnight, zone time. GMT is time zone zero.

Lines 9 through 15 - Initial Conditions

| | | | | | |
|---|---|---|---|---|---|
| 9 | | I | ICTYP | 2 | |
| 10 | | | IC | 126. 1 | |
| 11 | | | 2 | 31. 23 | |
| 12 | | | 3 | 89. | |
| 13 | | | 4 | 14. | |
| * 14 | | | 5 | 22600114 | |
| * 15 | | | 6 | 25117. 3 | |

Line 9 indicates the type of initial conditions entered in lines 10 thru 15.

For ICTYP = : IC's are:

1 Earth-centered inertial cartesian coordinates ($x, y, z, \dot{x}, \dot{y}, \dot{z}$ in feet and ft/sec - see paragraph 3. 1. 1. 1).

2 Spherical coordinates ($\alpha, \delta, \beta, A, r, v$ in degrees, feet and ft/sec - see paragraph 3. 1. 1. 2).

(*Line 14. If r is negative, it is interpreted as height above the earth's surface. *Line 15. If v is negative, circular velocity is computed and used.)

3 Orbital elements ($a, e, i, \Omega, \omega, \tau$ in feet, degrees, and minutes - see paragraph 3. 1. 1. 3).

4 The same as 2 above with longitude replacing right ascension.

5 No IC's are input. The last trajectory point of the last previous run is used.

6 No IC's are input. The corrected initial conditions from the last previous tracking run are used.

8 Same as 1, 2, and 3 but in units of earth radii, minutes, and radians The type is determined from the last previous tracking run, or from CPRAM (see line 37).

9 Same as 1 but in units of earth radii and earth radii/min.

0 No IC's are input. For a tracking run two RAE sets are used from the data to calculate a set of initial conditions (see paragraph 3.6.7).

5.2.1.2 Optional Input

Lines 16 through 20 - Drag

| | | | | |
|---|---|---|---|---|
| 16 | | DRAG | .01 | |
| 17 | I | 2 | | |
| 18 | | 3 | | |
| 19 | | 4 | | |
| 20 | | 5 | | |

Line 16 contains the drag parameter $\frac{C_D A}{W}$ in $ft^2/lb$.

Line 17 contains the atmosphere model specification.

| | | | |
|---|---|---|---|
| If DRAG(2) is: | 0 | use: | ARDC 59 Model |
| | 1 | | Lockheed Model |
| | 2 | | Paetzold Model II |
| | 3 | | Paetzold Model I |
| | 4 | | L. F. E. Model |

If line 17 is a 2, 3, or 4, lines 18 through 20 contain three quantities used in the Paetzold calculation.

Line 18 contains F - solar radio flux.

Line 19 contains $A_p$ - planetary magnetic index.

Line 20 contains g(a) - plasma intensity coefficient.

## Drag Table Option

Think of the drag parameter as two terms: $\frac{C_D A}{W} \times C'_D(M, h)$. $\frac{C_D A}{W}$ is a constant and can be differentially corrected by the use of the variational equation. $C'_D(M, h)$ is a function of Mach No. and/or altitude; it is then input as a table.

Normally $C'_D$ is set equal to 1, and with $\frac{C_D A}{W}$ input into location DRAG, TRACE operates as before. If the use of a table is desired, additional inputs must be made (marked with * below). The following is the use of the C block where the tables are stored.

| | | |
|---|---|---|
| | C(50) | $C'_D$(=1, or interpolated table value) |
| * | C(51) | if = 0, do not use tables<br>if negative, use Mach table only<br>if positive, use Mach table and altitude table |
| * | C(52) | altitude above which altitude table is used and below which Mach table is used. (Needed if C(51) is positive.) |
| | C(53) | (not used) |
| | C(54) | (used by interpolation routine) |
| * | C(55) - C(72) | altitude table. (Monotonic increasing stored as $alt_1$, $C'_D$, $alt_2$, $C'_D$ . . . $alt_n$, $C'_D$, 0, 0) |
| | C(73) | (not used) |
| | C(74) | (used by interpolation routine) |
| * | C(75) - C(100) | Mach No. table (stored as altitude table above). |

## Line 21 - Print Code

| | | | trajectory | residuals | partials | ATA | var. eq. | elements | constants | update | Z=0; B=90 | obs. times | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 21 | D | PRCDE | X | | | | | | X | | | | | |
| | | | ✓ | | | ✓ | ✓ | ✓ | | ✓ | | | | |

An X will cause the corresponding information to be output. (The boxes marked $\surd$ will be printed only at specified print times - see below.) See Section 5.3 for output examples.

Lines 22 through 28 - Print Time Vector

| | | | | | |
|---|---|---|---|---|---|
| | 22 | I | PRTIM | 0 | |
| n | 23 | I | 2 | 2 | |
| $t_o$ | 24 | | 3 | 0 | |
| $\Delta t_1$ | 25 | | 4 | 5 | |
| $t_1$ | 26 | | 5 | 90 | |
| $\Delta t_2$ | 27 | | 6 | 15 | |
| $t_2$ | 28 | | 7 | 1440 | |

Sequence of print times for output selected in PRCDE. There may be $n(n \leq 9)$ sets (line 23); for the $i^{th}$ set output is from $t_{i-1}$ to $t_i$ at intervals of $\Delta t_i$. All times are in minutes from midnight if PRTIM = 1; from epoch if PRTIM = 0. $\Delta t_i = 0$ means do not print in this interval. Additional cards may be inserted here if $3 \leq n \leq 9$.

Line 33 - Extra Body Perturbation

| | | | | | |
|---|---|---|---|---|---|
| | 33 | I | CTAPE | 7 | |

If CTAPE is non-zero, then other-body perturbations will be computed from coordinate tape on unit number CTAPE, using body selectors in INTEG and relative masses in C.

Line 34 - Eclipse Indication

| | | | | | |
|---|---|---|---|---|---|
| | 34 | I | XTAPE | | |

If XTAPE is non-zero, then the position of the sun will be determined at each print time from the coordinate tape on unit number XTAPE. An indication is then printed as to whether the sun is visible from the satellite.

5.2.2 Trajectory Only

5.2.2.1 Required Input

There is no required input in addition to the Basic Data.

5.2.2.2 Optional Input

Lines 37 through 39 - Variational Equation Partial Derivatives

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 37 | | D | CPRAM | 2 | | X | | | X | | | | | | |
| 38 | | D | DPRAM | | | | | | | X | X | | | | |
| 39 | | D | ΦPRAM | | | | | | | | | | | | |

An X in a box causes the variational equation for the corresponding parameter to be solved. The partial derivatives are printed out if the Variational Equation box is marked in PRCDE (see Line 21).

CPRAM - Initial condition parameters. The first box specifies the type of initial conditions. The succeeding boxes indicate the particular parameters desired. The boxes are ordered as follows:

| Type | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | $x$ | $y$ | $z$ | $\dot{x}$ | $\dot{y}$ | $\dot{z}$ | $t_o$ |
| 2 | $\alpha$ | $\delta$ | $\beta$ | $A$ | $r$ | $v$ | $t_o$ |
| 3 | $a$ | $e$ | $i$ | $\Omega$ | $\omega$ | $\tau$ | $t_o$ |

DPRAM and ΦPRAM - Differential equation parameters. The boxes are ordered as follows:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| DPRAM Drag | $\mu$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_{21}$* | $J_{31}$ | $J_{41}$ | $J_{22}$ | $J_{32}$ | $J_{42}$ |
| ΦPRAM $J_{33}$ | $J_{43}$ | $J_{44}$ | $\lambda_{21}$* | $\lambda_{31}$ | $\lambda_{41}$ | $\lambda_{22}$ | $\lambda_{32}$ | $\lambda_{42}$ | $\lambda_{33}$ | $\lambda_{43}$ | $\lambda_{44}$ |

*This list applies only if the full set is used. See the appendix for the ordering of shorter sets.

Line 42 - Trajectory Comparison Option

| | | | | |
|---|---|---|---|---|
| 41 | | IFLAG | | |
| 42 | I | 15 | 10 | |

If IFLAG(15) is

+ it contains the logical tape number of the tape, which is to contain the reference trajectory

0 a regular trajectory run is indicated

- it contains the logical tape number of the tape containing the reference trajectory. The reference trajectory is read in and differenced with the trajectory of the present case. These differences are resolved into the orbit plane system and are placed on the binary plot tape. The binary plot tape number is IFLAG(15) + 1.

Line 44 - Analytic Trajectory Option

| | | | | |
|---|---|---|---|---|
| 43 | | C | | |
| 44 | | 49 | | |

If C(49) is non-zero, the trajectory is not integrated. Instead, analytic approximation formulae are used to determine the trajectory (see paragraph 3.6.6). C(49) should be a positive integer; then the formulae are recomputed every $n^{th}$ orbit where n = C(49).

Table 1. TRACE - Basic Data and Trajectory Input

**X-1** 7090 INPUT DATA

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ____ KEYPUNCHED ____ VERIFIED ____ DATE ____ PAGE ____ OF ____

| 1 | 7 | 73 |
|---|---|---|
| H1 | | |
| H2 | | |
| | | |
| | | |

| SYMBOL | PR E (1, 19, 37, 55) | LOC. (2, 20, 38, 56) | VALUE (7, 25, 43, 61) | EXP. (17, 35, 53, 71) |
|---|---|---|---|---|
| 1 | D | ITIN | | |
| 2 | I | YEAR | | |
| 3 | I | MNTH | | |
| 4 | I | DAY | | |
| 5 | | TZNE | | |
| 6 | | HR | | |
| 7 | | MIN | | |
| 8 | | SEC | | |
| 9 | I | ICTYP | | |
| 10 | | IC | | |
| 11 | | 2 | | |
| 12 | | 3 | | |
| 13 | | 4 | | |
| 14 | | 5 | | |
| 15 | | 6 | | |
| 16 | | DRAG | | |
| 17 | I | 2 | | |
| 18 | | 3 | | |
| 19 | | 4 | | |
| 20 | | 5 | | |
| | | | trajectory, residuals, partials, ATA, var. eq., elements, constants, update, Z≠0;β=90, obs. times | |
| 21 | D | PRCDE | | |
| 22 | I | PRTIM | | |
| 23 | I | 2 | | |
| 24 | | 3 | | |
| 25 | | 4 | | |
| 26 | | 5 | | |
| 27 | | 6 | | |
| 28 | | 7 | | |
| 29 | | | | |
| 30 | | | | |
| 31 | | | | |
| 32 | | | | |
| 33 | I | CTAPE | | |
| 34 | I | XTAPE | | |
| | | | | |
| | | | | |

| SYMBOL | PR E (1, 19, 37, 55) | LOC. (2, 20, 38, 56) | VALUE (7, 25, 43, 61) | EXP. (17, 35, 53, 71) |
|---|---|---|---|---|
| 37 | D | CPRAM | | |
| 38 | D | DPRAM | | |
| 39 | D | ΦPRAM | | |
| | | | | |
| 41 | | IFLAG | | |
| 42 | I | 15 | | |
| 43 | | C | | |
| 44 | | 49 | | |
| | | | | |
| | | . | | |

AEROSPACE FORM 2423 REV 6-62

## Table 2. Basic Data and Trajectory Input Description

1: Itinerary. Specify order of computations.
1-Tracking data input, 2-Track,
3-Trajectory only,
4-Data generation,
5-Error analysis

2-8: Epoch

9: Type 1 2 3 4 of initial conditions

10-15: IC

| | | | |
|---|---|---|---|
| x | α | a | ℓ |
| y | δ | e | δ |
| z | β | i | β (ft-sec-deg) |
| $\dot{x}$ | A | Ω | A |
| $\dot{y}$ | r | ω | r |
| $\dot{z}$ | v | τ | v |

NOTE: If r is negative it is interpreted as height; if v is neg., circular velocity is computed and used.

16: $C_D A/W$. ($ft^2/lb$)

17: Atmosphere model specification
0 ARDC 59
1 Lockheed
2 Paetzold II
3 Paetzold I
4 L. F. E.

18: F - solar radio flux

19: $A_p$ - planetary magnetic index

20: g(a) - plasma intensity coeff.

21: X to print

| trajectory | residuals | partials | ATA Accum. | var. eq. | elements | constants | update | z=0;θ=90 | obs. times |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

22-28: Print at $t=t_o(\Delta t_1)\ t_1(\Delta t_2)\ldots(\Delta t_n)t_n$ in minutes from midnight if PRTIM=1, from epoch if 0.

23 - n (≤9), 24 - $t_o$, 25 - $\Delta t_1$,

26 - $t_1$, 27 - $\Delta t_2$, 28 - $t_2$, etc.

33: Extra body perturbation option and coordinate tape number.

34: Eclipse indication option and coordinate tape number.

37: Initial Condition Parameter Specification:

TYPE $\boxed{X}$ to specify:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | x | y | z | $\dot{x}$ | $\dot{y}$ | $\dot{z}$ | $t_o$ |
| 2 | α | δ | β | A | r | v | $t_o$ |
| 3 | a | e | i | Ω | ω | τ | $t_o$ |

38-39: Differential Eq. Parameter Specification:
X to specify:

| | $\frac{C_D A}{W}$ | μ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_{21}$* | $J_{31}$ |
|---|---|---|---|---|---|---|---|---|
| DPRAM | | | | | | | | |

| $J_{41}$ | $J_{22}$ | $J_{32}$ | $J_{42}$ |
|---|---|---|---|
| | | | |

| | $J_{33}$ | $J_{43}$ | $J_{44}$ | $L_{21}$* | $L_{31}$ | $L_{41}$ |
|---|---|---|---|---|---|---|
| OPRAM | | | | | | |

| $L_{22}$ | $L_{32}$ | $L_{42}$ | $L_{33}$ | $L_{43}$ | $L_{44}$ |
|---|---|---|---|---|---|
| | | | | | |

42: Trajectory comparison option and tape specification.

44: Analytic trajectory option.

* This list applies if ΦBJT = 4. If ΦBJT = 3 replace with shorter list $J_{21}$, $J_{31}$, $J_{22}$, $J_{32}$, $J_{33}$ and $L_{21}$, $L_{31}$, $L_{22}$, $L_{32}$, $L_{33}$. If ΦBJT = 2 replace with $J_{21}$, $J_{22}$ and $L_{21}$, $L_{22}$.

5.2.3 Tracking

5.2.3.1 Required Input

Lines 33 through 37 - Parameter Specification Boxes

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 33 | D | CPRAM | 1 | | X | | X | | | | | | | |
| 34 | D | DPRAM | | X | | | | | | X | X | X | | |
| 35 | D | ΦPRAM | | | | X | X | X | | | | | | |
| 36 | D | RPRAM | A | B | | | X | | | | | | | |
| 37 | D | | | | | | | | | | | | | |

An X in a box causes the corresponding parameter to be used in the differential correction solution.

CPRAM - Initial condition parameters. The first box specifies the type of initial condition. The succeeding boxes indicate the particular parameter desired. The boxes are ordered as follows:

| Type | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | $x$ | $y$ | $z$ | $\dot{x}$ | $\dot{y}$ | $\dot{z}$ | $t_o$ |
| 2 | $\alpha$ | $\delta$ | $\beta$ | $A$ | $r$ | $v$ | $t_o$ |
| 3 | $a$ | $e$ | $i$ | $\Omega$ | $\omega$ | $\tau$ | $t_o$ |

DPRAM and OPRAM - Differential equation parameters. The boxes are ordered as follows:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DPRAM | Drag | $\mu$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_{21}$* | $J_{31}$ | $J_{41}$ | $J_{22}$ | $J_{32}$ | $J_{42}$ |
| OPRAM | $J_{33}$ | $J_{43}$ | $J_{44}$ | $\lambda_{21}$* | $\lambda_{31}$ | $\lambda_{41}$ | $\lambda_{22}$ | $\lambda_{32}$ | $\lambda_{42}$ | $\lambda_{33}$ | $\lambda_{43}$ | $\lambda_{44}$ |

RPRAM - Radar parameters. The first two boxes of each line contain the station name. The succeeding boxes indicate the parameters desired. They are ordered as follows:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $L$ | $\ell$ | $A$ | $R$ | $A$ | $E$ | $\dot{R}$ | $T$ |

*This list applies if the full set is used. See Appendix A for explanation of standard set.

where:

$L$ = Latitude

$\ell$ = Longitude

$A$ = Altitude

$R$ = Range Bias

$A$ = Azimuth Bias

$E$ = Elevation Bias

$\dot{R}$ = Range Rate Bias

$R$ = Time Bias

Additional cards may be added for more radar stations. Note: The number of X's in CPRAM + DPRAM + OPRAM must be ≤ 15. The total number of X's must be ≤ 30.

Lines 41 through 46 - Bounds

| | | | | |
|---|---|---|---|---|
| 41 | | BNDS | 100 | |
| 42 | | 2 | 100 | |
| 43 | | 3 | . 1 | -5 |
| 44 | | 4 | | |
| 45 | | 5 | | |
| 46 | | 6 | | |

A bound must be entered for each parameter selected above in the same sequence. For each iteration of the differential correction process, the change in each parameter is:

a. Less (in absolute value) than the corresponding bound if this bound is positive.

b. Zero if the corresponding bound is zero.

c. Unrestricted if the corresponding bound is negative.

Lines 49 through 65 - Sigmas

| | | | | |
|---|---|---|---|---|
| 49 | | SIGMA | 100 | |
| 50 | | 2 | . 05 | |
| 51 | | 3 | . 05 | |
| 52 | | 12 | 200 | |
| 53 | | 13 | . 05 | |
| 54 | | 14 | . 1 | |

Sigmas are the weighting factors for the radar data. Entered here are a set of radar sigmas for R, A, E, $\dot{R}$, $\dot{P}$, $\dot{Q}$, P, Q, u, v, r in that order, in feet, degrees, and seconds. Ten sets may be entered, I = 0, 1, 2, ..., 9. This value of I is the one to be entered on the Station Location Card, column 5. Additional cards may be inserted here if necessary.

5.2.3.2 Optional Input

Line 29 - Tracking Termination Time

| | | | | |
|---|---|---|---|---|
| 29 | | 21 | | |

If PRTIM(21) is not zero, then only observations prior to this time (in minutes from midnight) will be used in the orbit determination.

Line 57 - Maximum Number of Iterations

| | | | | |
|---|---|---|---|---|
| 57 | I | MAXIT | 4 | |

If the differential correction process has not converged at the end of MAXIT iterations, the run will stop.

Line 58 through 60 - Data Tape Specifications

| | | | | |
|---|---|---|---|---|
| 58 | I | IBCDI | | |
| 59 | I | IBINI | | |
| 60 | I | IBINU | | |

IBCDI. If the radar observation and station location information is to come in on a BCD tape other than A3 (the normal FORTRAN system input tape), the tape number must be specified here. It may be any channel A tape not used by the system; only the numeric designation is required.

IBINI. If TRACE is to input a binary tape containing compacted radar data (produced by a previous run), IBINI must be non-zero. If IBINI $\leq$ 5, the tape is assumed to be on B5; if IBINI > 5, it is assumed to be the tape number (Channel B only).

IBINU. If IBINU is non-zero, TRACE will produce a binary tape containing the sorted processed radar observation data for later use. The same tape numbering convention holds as for IBINI above. If IBINU is non-zero, after a tape is produced IBINI will be set for the corresponding tape unit; successive cases of the same run will therefore not require that the observation input be repeated. (The tracking data input function must still be selected by ITIN in order to read the tape.)

<u>Line 61 - Refractivity</u>

| | | | | | |
|---|---|---|---|---|---|
| 61 | ■ | | REFR | | |

The equation used to correct elevation data is: $E = E' - n_{si} \text{ cotn } E'$ if $E' \geq 0.1$ radian, and $E = E' - \frac{1}{1000}\left(\frac{n_{si} \times 10^6}{12 + 1000E'} - \frac{80}{6 + 1000E'}\right)$ if $E' < 0.1$ radian, and $n_{si} \neq 0$.

$E'$ is the input elevation. The $n_{si}$ are read here, $i = 0, 1, 2, \ldots, 9$. This value of i is the one to be entered on the Station Location Card, column 6. Additional cards may be inserted here if necessary. Nominally, $n_{so} = 312.0 \times 10^{-6}$.

<u>Lines 63 and 64 - SΦS</u>

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 63 | ■ | D | SΦS | A | A | A | B | | | | | | | | |
| 64 | ■ | | | | | | | | | | | | | | |

SOS contains up to nine station names (2 blocks per name). For each of these stations, the root mean square (rms) and the square root of the sum of the squares (SOS) of the residuals after division by the radar sigmas are printed out.

Lines 65 and following - Constraint Matrix

| | | | | |
|---|---|---|---|---|
| 65 | I | KNST | 4 | |
| 66 | I | BLIST | 1 | |
| | I | | 1 | |
| | | | 1 | |
| | I | | 2 | |
| | I | | 2 | |
| | | | 1 | |
| | I | | 3 | |
| | I | | 3 | |
| | | | 1 | |
| | I | | 4 | |
| | I | | 4 | |
| | | | 1 | |
| | I | | 5 | |
| | I | | 1 | |
| | | | -1 | |
| | I | | 6 | |
| | I | | 2 | |
| | | | .5 | |
| | I | | 7 | |
| | I | | 5 | |
| | | | 1 | |

These quantities can be best explained by example. Assume that there are n parameters to be solved for $(p_1, p_2, \ldots, p_n) = p$. The ordering of the $p_i$ corresponds to the order of the X's in CPRAM, DPRAM, and RPRAM. Also assume that there are m linear constraints to be placed on these parameters. For example, if n = 6, m = 2, these might be $p_1 + p_5 = 6$, $p_2 - 2p_6 = 0$. Then KNST is equal to the number of effective (unconstrained) parameters, or 4(= n-m).

BLIST, the constraint matrix, is obtained as follows:

a. State the problem in the form $p = B\bar{p} + c$, where the $\bar{p}$ are the effective parameters. For the example given, this takes the form

$$\underbrace{\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix}}_{p} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & .5 & 0 & 0 \end{bmatrix}}_{B} \cdot \underbrace{\begin{bmatrix} \bar{p}_1 \\ \bar{p}_2 \\ \bar{p}_3 \\ \bar{p}_4 \\ \\ \\ \end{bmatrix}}_{\bar{p}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 6 \\ 0 \end{bmatrix}}_{c} \qquad (78)$$

b. Input the non-zero elements of the augmented (n+1) by (m+1) matrix

$$\begin{bmatrix} B & c \\ 0 & 1 \end{bmatrix}$$

where the element $b_{ij}$ is input as i, j, $b_{ij}$. The input for this example is shown above.

Lines 73 and 74 - Time Delay Correction

| | | | | |
|---|---|---|---|---|
| 73 | | C | | |
| 74 | | 3 | | |

If C(3) is non-zero, a time correction will be applied to the radar data. C(3) should contain ± the speed of light (earth radii/min).

Then, $t' = t + \frac{R}{C(3)}$ where R is the range.

Line 75 - Data Editing

| | | | | |
|---|---|---|---|---|
| 75 | | 13 | | |

If C(13) is non-zero, the radar data will be edited on the second and following iterations. Data points will be discarded with residuals greater than:

a. The input sigma times |C(13)| if C(13) is negative.

b. The statistical sigma from the previous iteration times C(13) if C(13) is positive. (A sigma is calculated for each station and data type.)

The above input should be followed by an END BASIC card. If the radar data is input from cards, Station Location Data and Radar Observation sheets should be filled out followed by an END DATA card. If the data is input from BCD or binary tape, the END DATA card follows the END BASIC card.

5.2.3.3 Station Location Data

| Column | |
|---|---|
| 1-2 | ST. Two letters that serve as identification for a station. No two stations should have the same symbol |
| 5 | Type of radar observation sigma to be applied to data from this station. The sets of sigmas input with the Basic Data are numbered (from 0 to 9) in the order in which they are read in. (See Line 49.) |
| 6 | Type of refractivity correction to be used for elevation readings from this station. Refractivities are numbered in the order they are input in the Basic Data. (See Line 61.) |
| 9-17 | North latitude of the station in degrees |
| 19-27 | East longitude of the station in degrees |
| 29-36 | Altitude of the station in feet |
| 39-39 (41-42) | If this station reports P or $\dot{P}$ data (Q or $\dot{Q}$), these columns contain the two letter symbols for the associated station(s) of the tracking configuration. Each such associated station must appear on a separate Station Location Card, but it is not necessary for columns 38-42 to be filled out on the latter. |

The last Station Location Card must be followed by a card with the letters TR in columns 1-2. There may be up to 50 stations entered.

### 5.2.3.4 Radar Observation Data

| | |
|---|---|
| ST | Station call letters, which must correspond to a Station Location Card |
| GMT | Number of hours to be added to the observation time to give Greenwich Mean Time |
| DAY | The values indicate the time of the corresponding observations |
| HR | |
| MIN | |
| SEC'S | |
| TY | Type of observation |

| TY | Col. 26-39 | Col. 41-54 | Col. 56-69 |
|---|---|---|---|
| 1 | Range | Azimuth | Elevation |
| 2 | Right Ascension | Declination | --- |
| 3 | L | M | N |
| 4 | Hour Angle | Declination | --- |
| 5 | $\Delta f$ | $\Delta t$ | --- |
| 6 | L1 | L2 | L3 |
| 7 | $\dot{R}$ | --- | --- |
| 8 | $\dot{P}$ | $\dot{Q}$ | |
| 9 | P | Q | |

The last Radar Observation Card must be followed by a card with the letters TR in columns 1-2. (An END DATA card must also follow if no nonstandard input is included.)

### 5.2.3.5 Flocking Option

For large numbers of radar observations (>1000) the data should be divided into "Flocks." Flocks may be of arbitrary size (but each <1000 observations). A control card with the letters TF in columns 1 and 2 is used to signal the end of a flock; any number of these may be placed among the observation cards. The last flock must still be terminated by a TR card.

There are two restrictions to be observed. First, the observations must be in partial chronological order. That is, every data time of a given flock must be later than all times in all previous flocks. Second, the Basic Data quantity IBINU (Line 60) must be specified so as to produce a compacted data tape, unless such a tape, produced on a previous run, is being used as input.

The mechanics of this option are as follows. The radar observations are read, sorted, processed, and written on tape, one flock at a time, by TRAIN. If more than one flock is found to be present, the differential correction process in MAIN reads the tape and computes residuals and the normal matrix for one flock at a time.

It is very strongly recommended that large sets of data be broken into flocks; looping and strange halts result from overreading observations with the program.

Table 3. TRACE - Tracking Input

**X-1** 7090 INPUT DATA

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ____ OF ____

| 1 | 7 | 73 |
|---|---|---|
| HI | | |
| H2 | | |
| | | |
| | | |

| SYMBOL | PR/E (1, 19, 37, 55) | LOC. (2, 20, 38, 56) | VALUE (7, 25, 43, 61) | EXP. (17, 35, 53, 71) |
|---|---|---|---|---|
| 1 | D | ITIN | | |
| 2 | I | YEAR | | |
| 3 | I | MNTH | | |
| 4 | I | DAY | | |
| 5 | | TZNE | | |
| 6 | | HR | | |
| 7 | | MIN | | |
| 8 | | SEC | | |
| 9 | I | ICTYP | | |
| 10 | | IC | | |
| 11 | | 2 | | |
| 12 | | 3 | | |
| 13 | | 4 | | |
| 14 | | 5 | | |
| 15 | | 6 | | |
| 16 | | DRAG | | |
| 17 | I | 2 | | |
| 18 | | 3 | | |
| 19 | | 4 | | |
| 20 | | 5 | | |
| | | | trajectory, residuals, partials, ATA, var. eq., elements, constants, update, z=0;8=90, obs.times | |
| 21 | D | PRCDE | | |
| 22 | I | PRTIM | | |
| 23 | I | 2 | | |
| 24 | | 3 | | |
| 25 | | 4 | | |
| 26 | | 5 | | |
| 27 | | 6 | | |
| 28 | | 7 | | |
| 29 | | 21 | | |
| 30 | | | | |
| 31 | | | | |
| 32 | | | | |
| 33 | D | CPRAM | | |
| 34 | D | DPRAM | | |
| 35 | D | ΦPRAM | | |
| 36 | D | RPRAM | | |

| SYMBOL | PR/E (1, 19, 37, 55) | LOC. (2, 20, 38, 56) | VALUE (7, 25, 43, 61) | EXP. (17, 35, 53, 71) |
|---|---|---|---|---|
| 37 | D | | | |
| 38 | D | | | |
| 39 | D | | | |
| 40 | D | | | |
| 41 | | BNDS | | |
| 42 | | | | |
| 43 | | | | |
| 44 | | | | |
| 45 | | | | |
| 46 | | | | |
| 47 | | | | |
| 48 | | | | |
| 49 | | SIGMA | | |
| 50 | | | | |
| 51 | | | | |
| 52 | | | | |
| 53 | | | | |
| 54 | | | | |
| 55 | | | | |
| 56 | | | | |
| 57 | I | MAXIT | | |
| 58 | I | IBCDI | | |
| 59 | I | IBINI | | |
| 60 | I | IBINU | | |
| 61 | | REFR | | |
| 62 | | | | |
| 63 | | SΦS | | |
| 64 | | | | |
| 65 | I | KNST | | |
| 66 | I | BLIST | | |
| 67 | | | | |
| 68 | | | | |
| 69 | | | | |
| 70 | | | | |
| 71 | | | | |
| 72 | | | | |
| 73 | | C | | |
| 74 | | 3 | | |
| 75 | | 13 | | |
| 76 | | | | |

AEROSPACE FORM 2423 REV 8-62

Table 4. Tracking Input Description

1: Itinerary. Specify order of computations.
1-Tracking data input, 2-Track,
3-Trajectory only,
4-Data generation,
5-Error analysis.

2-8: Epoch

9: Type 1 2 3 4 of initial conditions

10-15: IC

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| x | $\alpha$ | a | $\ell$ |
| y | $\delta$ | e | $\delta$ |
| z | $\beta$ | i | $\beta$ |
| $\dot{x}$ | A | $\Omega$ | A |
| $\dot{y}$ | r | $\omega$ | r |
| $\dot{z}$ | v | $\tau$ | v |

(ft-sec-deg)

NOTE: If r is negative it is interpreted as height; if v is neg., circular velocity is computed and used.

16: $C_DA/W$. ($ft^2/lb$)

17: Atmosphere Model Specification
0 ARDC 59
1 Lockheed
2 Paetzold II
3 Paetzold I
4 L. F. E.

18: F - solar radio flux
$A_p$ - planetary magnetic index
g(a) - plasma intensity coeff.

21: X to print: trajectory, residuals, partials, ATA Accum., var. eq., elements, constants, update, z=0;$\beta$=90, obs. times

22-28: Print at $t=t_o(\Delta t_1)t_1(\Delta t_2)\ldots(\Delta t_n)t_n$ in minutes from midnight if PRTIM=1, from epoch if 0.

23 - n ($\leq$9), 24 - $t_o$, 25 - $\Delta t_1$

26 - $t_1$, 27 - $\Delta t_2$, 28 - $t_2$, etc.

29: Fit only observations prior to $t_f$

33: Initial Condition Parameter Specification

| Type | X to solve for: | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | x | y | z | $\dot{x}$ | $\dot{y}$ | $\dot{z}$ | $t_o$ |
| 2 | $\alpha$ | $\delta$ | $\beta$ | A | r | v | $t_o$ |
| 3 | a | e | i | $\Omega$ | $\omega$ | $\tau$ | $t_o$ |

34-35: Differential Eq. Parameter Specification:
X to solve for:

DPRAM: $\frac{C_DA}{W}$ $\mu$ $J_2$ $J_3$ $J_4$ $J_5$ $J_{21}^*$ $J_{31}$ $J_{41}$ $J_{22}$ $J_{32}$ $J_{42}$

OPRAM: $J_{33}$ $J_{43}$ $J_{44}$ $L_{21}^*$ $L_{31}$ $L_{41}$ $L_{22}$ $L_{32}$ $L_{42}$ $L_{33}$ $L_{43}$ $L_{44}$

*or shorter lists depending on OBJT

36-40: Radar Parameter Specification.
X to solve for: ST L $\ell$ A R A E $\dot{R}$ T

| | | | |
|---|---|---|---|
| ST: | Station Symbol | R: | Range Bias |
| L: | Station Latitude | A: | Azimuth Bias |
| $\ell$: | Station Longitude | E: | Elevation Bias |
| A: | Station Altitude | $\dot{R}$: | Range Rate Bias |
| | | T: | Time Bias |

41-48: A bound must be provided for every parameter. For each iteration of the differential correction process, the change in each parameter (a) Is (in absolute value) less than the corresponding bound if said bound is positive, (b) Is zero if the corresponding bound is zero, (c) Is unrestricted if the corresponding bound is negative.

49-56: Sigmas
$(R, A, E, \dot{R}, \dot{P}, \dot{Q}, P, O, u, v, r)_l$
$l = 0, 1, 2, \ldots$
l is the sigma type as referred to by the Station Location Data.

57: Maximum Number of Iterations.

58-60: Radar Observation Data Tapes
58: BCD Input Tape if $\neq$ A3.
59: Non-zero, $\leq$ 5, if compacted data is to be input on B5.
60: Non-zero, $\leq$ 5, if compacted data is to be output on B5.

61: Refractivity.

63-64: The names of no more than nine stations. For each of these stations, the root mean square (RMS) and the square root of the sum of squares (SOS) of the residuals is printed out.

65: Number of effective (unconstrained) parameters to be solved for.

66-72: Constraint Matrix Input

74: Time delay correction.

75: Data editing option.

# Table 5. Radar Station Location Data

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COMPUTATION & MATHEMATICS CENTER

Table 5. Radar Station Location Data

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ____ OF ____

| 1 2 | 5 6 | 9 10 11 12 13 14 15 16 17 | 19 20 21 22 23 24 25 26 27 | 29 30 31 32 33 34 35 36 | 38 39 | 41 42 | 73 74 75 76 77 78 79 80 |
|---|---|---|---|---|---|---|---|
| S T | TYPE | LAT (NORTH)° | LONG (EAST)° | ALTITUDE (FT) | $P_{ST}$ $\dot{P}_{ST}$ | $Q_{ST}$ $\dot{Q}_{ST}$ | IDENTIFICATION |

ST is the station identification code.

P, $\dot{P}$ ST must be input if either or both of the two quantities are selected on the "Data Specification II" Load Sheet. (Likewise for Q, $\dot{Q}$ ST)

AEROSPACE FORM

# Table 6. Radar Observation Data

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Table 6. Radar Observation Data

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ______ OF ______

| ST | MT | MO | DAY | HR | MN | SEC'S | TY | R, RA, L, HA, Δf, L1, Degrees, ft, ft/sec | R, P, P MIN | SEC | A, D, M, D, Δt, L2, R, Q, Q Degrees | MIN | SEC | E, __, N, __, __, L3, Degrees | MIN | SEC | IDENT. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | |

5.2.4 Data Generation

5.2.4.1 Required Input

The only required input in addition to the Basic Data is the station location data and data on Data Specification Sheets I and II. These will be described below.

5.2.4.2 Optional Input

Line 34 - Rise and Set Times

| | | | | |
|---|---|---|---|---|
| | | IFLAG | | |
| 34 | I | 6 | | |

If IFLAG(6) is

0 all data will be printed (see sample output paragraph 5.4.5).

1 rise and set times only will be printed; Data Specification II not necessary.

Rise and sets are at minimum elevation angle entered on Data Specification Sheet I.

Line 35 - Input Control for Multiple Cases

| | | | | |
|---|---|---|---|---|
| 35 | I | 7 | | |

Station Location cards and Data Specification cards are always read when a 4 or a 5 is first encountered in the ITIN list. In each following instance in the same ITIN sequence:

if IFLAG(7) is

0 Neither Station Location or Data Specification cards are input (same as previous case).

1 Data Specification is input, but Station Locations are not.

-1 Both Station Location and Data Specification are input.

Line 36 - Observation Tape Generation

| | | | | |
|---|---|---|---|---|
| 36 | I | ETAPE | 7 | |

If ETAPE is non-zero, then a BCD radar observation tape will be generated on tape unit number ETAPE. The tape format will be that of the tracking input data.

Lines 37 through 52 - Noise

| | | | | |
|---|---|---|---|---|
| 37 | B | NØISE | 0 1 2 3 4 0 0 5 | |
| 38 | D | RPRAM | A A ③ X X X | |
| 39 | D | | ③ | |
| 40 | D | | ③ | |
| 41 | | PBIAS | .001 | |
| | | 2 | 30 | |
| | | 3 | -.005 | |
| | | 4 | .001 | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| 49 | | SIGMA | 50 | |
| | | 2 | .01 | |
| | | 3 | .01 | |
| | | 4 | | |

If NOISE is non-zero, the observations on the above tape and in the printed data generation output will contain normally distributed random noise with mean value given in PBIAS (Lines 41-ff) and standard deviations given in SIGMA (Lines 49-ff). (NOISE starts random number generator.)

If bias noise is to be used, RPRAM (Lines 38 through 40) contain the radar observations to be biased. The first two boxes of each line contain the station name. The succeeding boxes indicate the parameters desired. They are ordered as follows:

| | | | | | |
|---|---|---|---|---|---|
| 38 | D | RPRAM | | ③ R A E $\dot{R}$ T | |

where:

R = Range Bias

A = Azimuth Bias

E = Elevation Bias

$\dot{R}$ = Range Rate Bias

T = Time Bias

Additional cards may be added for more radar stations.

PBIAS (Lines 41 - ff.) contain the bias to be added. The ordering is the same as the X's in the RPRAM boxes above.

SIGMA (Lines 49 - ff.) contain the standard deviation of the random noise to be added. Entered here are a set of radar sigmas consisting of values for R, A, E, $\dot{R}$, $\dot{P}$, $\dot{Q}$, P, Q, u, v, r in that order, in feet, degrees, and seconds. I sets may be entered, I = 0, 1, 2, . . . , 9. This value of I is the one to be entered on the Station Location Card, column 5. Additional cards may be inserted here if necessary.

Line 57 - Refractivity

| | | | | |
|---|---|---|---|---|
| 57 | | REFR | | |

The computed elevation is altered to account for refraction using the following formula:

$$E' = E + n_{si} \cot E \text{ if } E \geq 0.1 \text{ radian} \tag{79}$$

and

$$E' = E + \frac{1}{1000}\left(\frac{n_{si} \times 10^6}{12 + 1000E} - \frac{80}{6 + 1000E}\right) \text{ if } E < 0.1 \text{ radian and } n_{si} \neq 0. \tag{80}$$

E is the computed elevation. The $n_{si}$ are read here, i = 0, 1, 2, . . . 9. This value of i is the one to be entered on the Station Location Card, column 6. Additional cards may be inserted here if necessary. Nominally, $n_{so} = 312.0 \times 10^{-6}$.

Line 65 - Observational Variances

| | | | |
|---|---|---|---|
| 65 | D | CPRAM | |
| 65 | D | DPRAM | |
| 67 | D | ΦPRAM | |

| | | |
|---|---|---|
| 69 | ATA | |
| 70 | 501 | |

If the standard deviations in the six observational quantities R, A, E, $\dot{R}$, $\dot{A}$, $\dot{E}$, are desired, then

a. The appropriate box must be checked on Data Specification Sheet II,

b. A covariance matrix for trajectory parameters must be supplied in lower triangular form beginning at ATA(501)

c. The corresponding parameters must be indicated (as on lines 37-39 of the Basic Data Input sheet) in the CPRAM, DPRAM, and OPRAM boxes.

The above input should be followed by an END BASIC card and an END DATA card. Station Location cards and Data Specification cards follow.

5.2.4.3 Station Location Data

Column

1-2 ST. Two letters, which serve as identification for a station. No two stations should have the same symbol.

5 Type of radar observation sigma to be applied to data from this station
The sets oi sigmas input with the Basic Data are numbered (from 0 to 9) in the order in which they are read in. (See Line 49.)

6 Type of refractivity correction to be used for elevation readings from this station.
Refractivities are numbered in the order they are input in the Basic Data. (See above.)

9-17 North latitude of the station in degrees

19-27 East longitude of the station in degrees

| | |
|---|---|
| 29-36 | Altitude of the station in feet |
| 38-39 (41-42) | If this station reports P or $\dot{P}$ data (Q or $\dot{Q}$), these columns contain the two letter symbols for the associated station(s) of the tracking configuration. Each such associated station must appear on a separate Station Location Card, but it is not necessary for columns 38-42 to be filled out on the latter. |

The last Station Location Card must be followed by a card with the letters TR in columns 1 and 2. There may be up to 50 stations entered.

5.2.4.4 Data Specification

| Column | Load Sheet I |
|---|---|
| 1-2 | Station Call Letters<br>These must correspond to the letters on some Station Location card |
| 9-16 | Interval, in minutes, at which data for this station is to be generated; also testing interval for Rise-Set-only option |
| 18-23 | Minimum elevation at which the vehicle is visible |
| 25-30 | Maximum elevation at which the vehicle is visible<br>(Zero value will be set to $90^{\circ}$) |
| 32-40 | Maximum range (in nautical miles) to which vehicle is visible<br>(Zero value causes this test to be ignored) |
| 51-58 | Start time, from midnight of start date<br>(Zero value implies epoch is start time)<br>51-52 days<br>54-55 hours<br>57-58 minutes |
| 60-67 | Stop time, from midnight of start date<br>60-61 days<br>63-64 hours<br>66-67 minutes |

The last card of this type must be followed by a card with the letters TR in columns 1 and 2.

| Column | Load Sheet II (Note: This is not used for the Rise-Set-only option) |
|---|---|
| 1-2 | Station Call Letters<br>These must correspond to the letters on some card from Load Sheet I |

| | |
|---|---|
| 7-25 | An X in the appropriate column will cause the quantity listed above that column to be output. (Only columns 7 through 14 will be written on ETAPE if that option is used.) (See Line 36.) |
| 7 | Range (n mi but written on ETAPE in ft) |
| 8 | Azimuth (deg) |
| 9 | Elevation (deg) |
| 10 | Range Rate (ft/sec) |
| 11-14 | P Dot, Q Dot, P, Q - Doppler Data |
| 15 | Azimuth Rate (deg/min) |
| 16 | Elevation Rate (deg/min) |
| 17 | Range Acceleration (ft/sec$^2$) |
| 18 | Mutual Visibility<br>Output will be a list of numbers of the stations that are visible at the output time.<br>(Stations are numbered in the order they are input on Station Location cards.) There is a maximum of 8 stations. |
| 19 | Latitude of vehicle (deg) |
| 20 | Longitude of vehicle (deg) |
| 21 | Surface Range from station to subvehicle point (n mi) |
| 22 | Altitude of vehicle (n mi) |

The following options require special input prior to the END cards:

| | |
|---|---|
| 23 | Doppler Rate = K × Range Rate<br>K is input into C(29) |
| 24 | Look Angle<br>This is the angle between an axis in the vehicle and the line of sight from the station to the vehicle. The direction cosines of the vehicle axis must be in C(37), C(38), and C(39). These may be input as constant, or the user may provide a subroutine called FANG that computes the direction cosines at each output point. |
| 25 | Observation Variances<br>The standard deviations of the six observational quantities R, A, E, $\dot{R}$, $\dot{A}$, $\dot{E}$, are output. These are based on a variance-covariance matrix for trajectory parameters. The matrix is input beginning in ATA(501) in lower triangular form (see Line 69). Corresponding parameters must be indicated in the CPRAM, DPRAM, and OPRAM boxes (see Line 65). |

Table 7. TRACE - Data Generation Input

**X-1** 7090 INPUT DATA

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PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ____ OF ____

| 1 | 7 | 73 |
|---|---|---|
| H1 | | |
| H2 | | |
| | | |
| | | |

| SYMBOL | PRE | LOC. | VALUE | EXP. |
|---|---|---|---|---|
| 1 | D | ITIN | | |
| 2 | I | YEAR | | |
| 3 | I | MNTH | | |
| 4 | I | DAY | | |
| 5 | | TZNE | | |
| 6 | | HR | | |
| 7 | | MIN | | |
| 8 | | SEC | | |
| 9 | I | ICTYP | | |
| 10 | | IC | | |
| 11 | | 2 | | |
| 12 | | 3 | | |
| 13 | | 4 | | |
| 14 | | 5 | | |
| 15 | | 6 | | |
| 16 | | DRAG | | |
| 17 | I | 2 | | |
| 18 | | 3 | | |
| 19 | | 4 | | |
| 20 | | 5 | | |
| | | | trajectory, residuals, partials, ATA, var. eq., elements, constants, update, z=0;β=90, obs. times | |
| 21 | D | PRCDE | | |
| 22 | I | PRTIM | | |
| 23 | I | 2 | | |
| 24 | | 3 | | |
| 25 | | 4 | | |
| 26 | | 5 | | |
| 27 | | 6 | | |
| 28 | | 7 | | |
| 29 | | | | |
| 30 | | | | |
| 31 | | | | |
| 32 | | | | |
| 33 | | IFLAG | | |
| 34 | I | 6 | | |
| 35 | I | 7 | | |
| 36 | I | ETAPE | | |

| SYMBOL | PRE | LOC. | VALUE | EXP. |
|---|---|---|---|---|
| 37 | B | NØISE | | |
| 38 | D | RPRAM | 3 | |
| 39 | D | | 3 | |
| 40 | D | | 3 | |
| 41 | | PBIAS | | |
| 42 | | 2 | | |
| 43 | | 3 | | |
| 44 | | 4 | | |
| 45 | | | | |
| 46 | | | | |
| 47 | | | | |
| 48 | | | | |
| 49 | | SIGMA | | |
| 50 | | 2 | | |
| 51 | | 3 | | |
| 52 | | 4 | | |
| 53 | | | | |
| 54 | | | | |
| 55 | | | | |
| 56 | | | | |
| 57 | | REFR | | |
| 58 | | | | |
| 59 | | | | |
| 60 | | | | |
| 61 | | | | |
| 62 | | | | |
| 63 | | | | |
| 64 | | | | |
| 65 | D | CPRAM | | |
| 66 | D | DPRAM | | |
| 67 | D | ΦPRAM | | |
| 68 | | | | |
| 69 | | ATA | | |
| 70 | | 501 | | |
| 71 | | | | |
| 72 | | | | |
| 73 | | | | |
| 74 | | | | |
| 75 | | | | |
| 76 | | | | |

AEROSPACE FORM 2423 REV 6-62

## Table 8. Data Generation Input Description

1: Itinerary. Specify order of computations.
1-Tracking data input, 2-Track,
3-Trajectory only,
4-Data generation,
5-Error analysis

2-8: Epoch

9: Type 1 2 3 4 of initial contitions
10-15: IC

| 1 | 2 | 3 | 4 | |
|---|---|---|---|---|
| x | $\alpha$ | a | $\ell$ | |
| y | $\delta$ | e | $\delta$ | |
| z | $\beta$ | i | $\theta$ | (ft-sec-deg) |
| $\dot{x}$ | A | $\Omega$ | A | |
| $\dot{y}$ | r | $\omega$ | r | |
| $\dot{z}$ | v | $\tau$ | v | |

NOTE: If r is negative it is interpreted as height; if v is neg., circular velocity is computed and used.

16: $C_DA/W$ (ft$^2$/lb)

17: Atmosphere Model Specification

0 ARDC 59
1 Lockneed
2 Paetzold II
3 Paetzold 1
4 L. F. E.

18: F - solar radio flux

19: $A_p$ - planetary magnetic index

20: g(a) - plasma intensity coeff.

trajectory, residuals, partials, ATA Accum., Var. Eq., elements, constants, update, $z=0;\beta=90$, obs. times

21: X to print | | | | | | | | | | | |

22-28: Print at $t=t_o(\Delta t_1)t_1(\Delta t_2)\ldots(\Delta t_n)t_n$ in minutes from midnight if PRTIM=1, from epoch if 0.

23 - n ($\leq 9$), 24 - $t_o$, 25 - $\Delta t_1$

26 - $t_1$, 27 - $\Delta t_2$, 28 - $t_2$, etc.

34: If non-zero, rise and set times only will be generated.

35: Input control for station location and ephemeris cards.

36: Tape number of BCD radar station and observation tape. (If zero, no tape is generated.)

37: If non-zero, the above tape will contain normally distributed random noise (mean values in PBIAS and standard deviations in SIGMA; RPRAM specifies the bias parameters).

38-40: Radar Parameter Specification

| | S | T | | R | A | E | $\dot{R}$ | T |
|---|---|---|---|---|---|---|---|---|
| X to specify | | | 3 | | | | | |

ST: Station Symbol
3 blank spaces

R: Range Bias
A: Azimuth Bias
E: Elevation Bias
$\dot{R}$: Range Rate Bias
T: Time Bias

41-48: Contains the mean values of the biases to be added to the observations specified in RPRAM.

49-56: Contains the standard deviations of the random noise to be added.

57-64: Refractivity.

65: Initial Condition Parameter Specification:
TYPE $\boxed{X}$ to specify:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | x | y | z | $\dot{x}$ | $\dot{y}$ | $\dot{z}$ | $t_o$ |
| 2 | $\alpha$ | $\delta$ | $\beta$ | A | r | v | $t_o$ |
| 3 | a | e | i | $\Omega$ | $\omega$ | $\tau$ | $t_o$ |
| | | | | | | | |

66-67: Differential Eq. Parameter Specification:
X to specify:

| | $\frac{C_DA}{W}$ | $\mu$ | $J_2$ | $\dot{J}_3$ | $J_4$ | $J_5$ | $J_{21}^*$ | $J_{31}$ |
|---|---|---|---|---|---|---|---|---|
| DPRAM | | | | | | | | |

| $J_{41}$ | $J_{22}$ | $J_{32}$ | $J_{42}$ |
|---|---|---|---|
| | | | |

| | $J_{33}$ | $J_{43}$ | $J_{44}$ | $L_{21}^*$ | $L_{31}$ | $L_{41}$ |
|---|---|---|---|---|---|---|
| OPRAM | | | | | | |

| $L_{22}$ | $L_{32}$ | $L_{42}$ | $L_{33}$ | $L_{43}$ | $L_{44}$ |
|---|---|---|---|---|---|
| | | | | | |

69-76: Covariance matrix in lower triangular form for parameters selected above.

*This list applies if $\Phi$BJT = 4. If $\Phi$BJT = 3 replace with shorter list $J_{21}$, $J_{31}$, $J_{22}$, $J_{32}$, $J_{33}$ and $L_{21}$, $L_{31}$, $L_{22}$, $L_{32}$, $L_{33}$. If $\Phi$BJT = 2 replace with $J_{21}$, $J_{22}$ and $L_{21}$, $L_{22}$.

## Table 9. Radar Station Location Data

AEROSPACE CORPORATION
COMPUTATION & MATHEMATICS CENTER

Table 9. Radar Station Location Data

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ______ OF ______

| 1 2 | 5 8 | 9 10 | 11 12 13 14 15 16 17 | 19 20 | 21 22 23 24 25 26 27 | 29 30 | 31 32 33 34 35 36 | 38 39 | 41 42 | 73 74 75 76 77 78 79 80 |
|---|---|---|---|---|---|---|---|---|---|---|
| S T | TYPE | | LAT (NORTH)° | | LONG (EAST)° | | ALTITUDE(FT) | $P\ \dot{P}$ S T | $Q\ \dot{Q}$ S T | IDENTIFICATION |

ST is the station identification code.
$P, \dot{P}$ ST must be input if either or both of the two quantities are selected on the "Ephemeris or Simulation Data Specification II" Load Sheet.
(Likewise for $Q, \dot{Q}$ ST)

## Table 8. Data Generation Input Description

1: Itinerary. Specify order of computations.
1-Tracking data input, 2-Track,
3-Trajectory only,
4-Data generation,
5-Error analysis

2-8: Epoch

9: Type 1 2 3 4 of initial contitions

10-15: IC

| Type | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| IC | $x$ | $\alpha$ | $a$ | $\ell$ | |
| | $y$ | $\delta$ | $e$ | $\delta$ | |
| | $z$ | $\beta$ | $i$ | $\theta$ | (ft-sec-deg) |
| | $\dot{x}$ | $A$ | $\Omega$ | $A$ | |
| | $\dot{y}$ | $r$ | $\omega$ | $r$ | |
| | $\dot{z}$ | $v$ | $\tau$ | $v$ | |

NOTE: If r is negative it is interpreted as height; if v is neg., circular velocity is computed and used.

16: $C_D A/W$ ($ft^2/lb$)

17: Atmosphere Model Specification

0 ARDC 59
1 Lockheed
2 Paetzold II
3 Paetzold I
4 L. F. E.

18: F - solar radio flux

19: $A_p$ - planetary magnetic index

20: g(a) - plasma intensity coeff.

21: X to print

| trajectory | residuals | partials | ATA Accum. | Var. Eq. | elements | constants | update | z=0;β=90 | obs. times |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

22-28: Print at $t=t_o(\Delta t_1)t_1(\Delta t_2)\ldots(\Delta t_n)t_n$ in minutes from midnight if PRTIM=1, from epoch if 0.

23 - n ($\leq 9$), 24 - $t_o$, 25 - $\Delta t_1$

26 - $t_1$, 27 - $\Delta t_2$, 28 - $t_2$, etc.

34: If non-zero, rise and set times only will be generated.

35: Input control for station location and ephemeris cards.

36: Tape number of BCD radar station and observation tape. (If zero, no tape is generated.)

37: If non-zero, the above tape will contain normally distributed random noise (mean values in PBIAS and standard deviations in SIGMA; RPRAM specifies the bias parameters).

38-40: Radar Parameter Specification

| | S | T | ③ | R | A | E | Ṙ | T |
|---|---|---|---|---|---|---|---|---|
| X to specify | | | | | | | | |

ST: Station Symbol
③ blank spaces

R: Range Bias
A: Azimuth Bias
E: Elevation Bias
Ṙ: Range Rate Bias
T: Time Bias

41-48: Contains the mean values of the biases to be added to the observations specified in RPRAM.

49-56: Contains the standard deviations of the random noise to be added.

57-64: Refractivity.

65: Initial Condition Parameter Specification:
TYPE $\boxed{X}$ to specify:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | $x$ | $y$ | $z$ | $\dot{x}$ | $\dot{y}$ | $\dot{z}$ | $t_o$ |
| 2 | $\alpha$ | $\delta$ | $\beta$ | $A$ | $r$ | $v$ | $t_o$ |
| 3 | $a$ | $e$ | $i$ | $\Omega$ | $\omega$ | $\tau$ | $t_o$ |
| | | | | | | | |

66-67: Differential Eq. Parameter Specification:
X to specify:

| | $\frac{C_D A}{W}$ | $\mu$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_{21}$* | $J_{31}$ |
|---|---|---|---|---|---|---|---|---|
| DPRAM | | | | | | | | |

| $J_{41}$ | $J_{22}$ | $J_{32}$ | $J_{42}$ |
|---|---|---|---|
| | | | |

| | $J_{33}$ | $J_{43}$ | $J_{44}$ | $L_{21}$* | $L_{31}$ | $L_{41}$ |
|---|---|---|---|---|---|---|
| OPRAM | | | | | | |

| $L_{22}$ | $L_{32}$ | $L_{42}$ | $L_{33}$ | $L_{43}$ | $L_{44}$ |
|---|---|---|---|---|---|
| | | | | | |

69-76: Covariance matrix in lower triangular form for parameters selected above.

*This list applies if ΦBJT = 4. If ΦBJT = 3 replace with shorter list $J_{21}$, $J_{31}$, $J_{22}$, $J_{32}$, $J_{33}$ and $L_{21}$, $L_{31}$, $L_{22}$, $L_{32}$, $L_{33}$. If ΦBJT = 2 replace with $J_{21}$, $J_{22}$ and $L_{21}$, $L_{22}$.

## Table 10. Data Generation or Error Analysis Data Specification I

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Table 10. Data Generation or Error Analysis Data Specification I

GRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ____ OF ____

| 1 2 | 9 10 | 11 12 13 14 15 16 | 18 19 20 | 21 22 23 | 25 26 27 28 29 30 | 32 33 34 35 36 37 38 39 40 | 51 52 53 54 55 56 57 58 | 60 | 61 62 63 64 65 66 67 |
|---|---|---|---|---|---|---|---|---|---|
| ST | IN | TERVAL | MIN | . EL | MAX , EL | MAX . RANGE | START TIME | S | TOP TIME |
| | ( | MIN ) * | ( DE | G) * | ( DEG ) * | ( N.MI LES ) * | DA HR MN | D | A HR MN |

ST is the station identification code.

START and STOP times are from midnight of epoch.

If start time is left zero, epoch will be defined as start time.

An * following the unit value means a decimal point is necessary for that quantity.

## Table 11. Data Generation Data Specification II

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Table 11. Data Generation Data Specification II

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ______ OF ______

| Columns | Entry |
|---|---|
| 1–2 | ST |
| 7 | Range |
| 8 | Azimuth |
| 9 | Elevation |
| 10 | Range Rate |
| 11 | P Dot |
| 12 | Q Dot |
| 13 | P |
| 14 | Q |
| 15 | Azm. Rate |
| 16 | Elev. Rate |
| 17 | R Dot Dot |
| 18 | Mutual Vis |
| 19 | Latitude |
| 20 | Longitude |
| 21 | Surface Range |
| 22 | Height |
| 23 | Doppler Rate |
| 24 | Look Angle |
| 25 | Obs. Variances |
| 73–80 | IDENT. |

ST is the station identification code.

An X in a column will cause the appropriate quantity to be output.

If P, P dot, Q, or Q dot are selected, the specification of an additional station identification code is necessary. (See Radar Location Data Input Sheet)

AEROSPACE FORM

### 5.2.5 Error Analysis

#### 5.2.5.1 Required Input

Lines 29 through 36 - Parameter Specification Boxes

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 29 | D | CPRAM | 1 | | P | | P | Q | P | | | | | |
| 30 | D | DPRAM | | | Q | | | | P | | | | Q | |
| 31 | D | ΦPRAM | | | | | Q | | Q | | | | | |
| 32 | D | RPRAM | A | B | | | P | | | | Q | | | Q |
| 33 | D | | | | | | | | | | | | | |
| 34 | | | | | | | | | | | | | | |
| 35 | | | | | | | | | | | | | | |
| 36 | | | | | | | | | | | | | | |

A P in a box causes the corresponding parameter to be used as a "P" parameter. A Q in a box specifies that the associated parameter is a "Q" parameter.

- CPRAM - Initial condition parameters. The first box specifies the type of initial condition. The succeeding boxes indicate the particular parameter desired. The boxes are ordered as follows:

| Type CPRAM | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | $x$ | $y$ | $z$ | $\dot{x}$ | $\dot{y}$ | $\dot{z}$ | $t_o$ |
| 2 | $\alpha$ | $\delta$ | $\beta$ | $A$ | $r$ | $v$ | $t_o$ |
| 3 | $a$ | $e$ | $i$ | $\Omega$ | $\omega$ | $\tau$ | $t_o$ |

- DPRAM and $\phi$PRAM - Differential equation parameters. The boxes are ordered as follows:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DPRAM | Drag | $\mu$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_{21}$* | $J_{31}$ | $J_{41}$ | $J_{22}$ | $J_{32}$ | $J_{42}$ |
| $\phi$PRAM | $J_{33}$ | $J_{43}$ | $J_{44}$ | $\lambda_{21}$* | $\lambda_{31}$ | $\lambda_{41}$ | $\lambda_{22}$ | $\lambda_{32}$ | $\lambda_{42}$ | $\lambda_{33}$ | $\lambda_{43}$ | $\lambda_{44}$ |

- RPRAM - Radar parameters. The first two boxes of each line contain the station name. The succeeding boxes indicate the particular parameter desired. They are ordered as follows:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| RPRAM | | | $L$ | $\ell$ | $A$ | $R$ | $A$ | $E$ | $\dot{R}$ | $T$ | $u$ | $v$ |

where

$L$ = Latitude

$\ell$ = Longitude

$A$ = Altitude

$R$ = Range Bias

$A$ = Azimuth Bias

$E$ = Elevation Bias

$\dot{R}$ = Range Rate Bias

$T$ = Time Bias

$u$ = "u" Bias

$v$ = "v" Bias

*This list applies only if the full set is used. See the appendix for explanation of standard set.

Additional cards may be added for more radar stations. Note: The number P's plus the number of Q's in CPRAM + DPRAM + ΦPRAM must be ≤ 15. The total number of P's plus Q's must be ≤ 30.

Lines 37 through 44 - Sigmas

| | | | | |
|---|---|---|---|---|
| 37 | | SIGMA | 100 | |
| 38 | | 2 | .05 | |
| 39 | | 3 | .05 | |
| 40 | | 10 | 50. | |
| 41 | | 12 | 120 | |
| 42 | | 13 | .06 | |
| 43 | | 14 | .06 | |
| 44 | | 21 | 60. | |

Sigmas are the weighting factors for the radar observation partials. Each sigma is a standard deviation for the particular observation type and station. A set of radar sigmas consists of values for R, A, E, $\dot{R}$, $\dot{P}$, $\dot{Q}$, P, Q, u, v, r; in that order, in feet, degrees, and seconds, I sets may be entered, I = 0, 1, 2, . . . , 9. This value of I is the one to be entered on the Station Location Card, column 5. Additional cards may be inserted here if necessary.

5.2.5.2 Optional Input

Line 45 - Covariance Output Specifications

(If any covariance matrix output is desired the eighth box, labeled "update," of PRCDE must be checked.)

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45 | | D | PRCΦV | X | | D | X | | | D | D | | X | |

An X in a box specifies that the whole covariance matrix be output. A D in a box causes only the square roots of the diagonals to be output. The boxes are ordered as follows:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (Inverse) | (Cartesian) | (Orbit Plane) | (Polar) | (Element) | (Period, Apogee, Perigee) | (Inverse)* | (Cartesian)* | (Orbit Plane)* | (Polar)* | (Element)* | (Period, Apogee, Perigee)* |
| PRCOV | $C(P_o)$ | $C(X)_1$ | $C(\Xi)_1$ | $C(R)_1$ | $C(E)_1$ | $C(T)_1$ | $C(P_2)$ | $C(X)_2$ | $C(\Xi)_2$ | $C(R)_2$ | $C(E)_2$ | $C(T)_2$ |

Line 46 - Additional Option Box

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 46 | D | ΦPBΦX | X | X | 1 | | |

An explanation of the purpose of each box follows:

ΦPBΦX | A | B | C |

A. If "A" box contains an X the $\partial P/\partial Q$ will be printed.

B. If "B" box contains an X the $A^T A$ print will be omitted.

C. If "C" box contains:

1, the $A^T A$ will be punched.

2, the partitioned $A_P{}^T A_P$ matrix will be punched.

Lines 49 through 60 - C(Q) Covariance Matrix Input

| | | | | |
|---|---|---|---|---|
| 49 | | CΦVQ | .01 | |
| 50 | | 2 | .01 | |
| 51 | | 3 | .01 | |
| 52 | | 4 | 5. | |
| 53 | | 5 | 5. | |
| 54 | | 6 | 2500. | |
| 55 | | | | |
| 56 | | | | |

*These covariance matrices include the "Q" effects.

COVQ contains the variance-covariance matrix of the "Q" parameters, which are specified in CPRAM, DPRAM, QPRAM. and RPRAM. The matrix is input in lower triangular form. For example, if there were 3 Q's specified, the matrix would have the form:

$$\begin{array}{lll} \sigma_{11}^2 & & \\ \sigma_{12}^2 & \sigma_{22}^2 & \\ \sigma_{13}^2 & \sigma_{23}^2 & \sigma_{33}^2 \end{array}$$

This matrix is input into COVQ in the order:

$$\begin{array}{lll} 1 & & \\ 2 & 3 & \\ 4 & 5 & 6 \end{array}$$

Lines 61 and 62 - Station Location and Data Specification Input Option for Multiple Cases[*]

| | | | | | |
|---|---|---|---|---|---|
| 61 | | | IFLAG | | |
| 62 | | I | 7 | 1 | |

If IFLAG(7) is:

- -1 Input all Station Location and Data Specification Cards
- 0 No input - values are the same as previous case
- 1 Input Data Specification cards only; station locations are the same as previous case.

[*]This option applies to "multiple" cases (that is, ITIN = 555 ...), but not to "stacked" cases (successive cases for each of which ITIN = 5).

Line 63 - $A^TA$ or $(A^TA)^{-1}$ Input Option

| 63 | I | 15 | 1 | |
|---|---|---|---|---|

If IFLAG (15) is:

| | |
|---|---|
| -1 | Hold $A^TA$ from previous case |
| 0 | No $A^TA$ or inverse input |
| 1 | Input $A^TA$ into ATA area, as augmented upper triangular matrix |
| 2 | Input inverse into ATA (501) area, as lower triangular matrix. |

5.2.5.3 Station Location Data

| Column | |
|---|---|
| 1-2 | ST<br>Two letters, which serve as identification for a station.<br>No two stations should have the same symbol |
| 5 | Type of radar observation sigma to be applied to data from this station<br>The sets of sigmas input with the Basic Data are numbered (from 0 to 9) in the order in which they are read in (see Lines 37 through 44). |
| 6 | Not used for Error Analysis |
| 9-17 | North latitude of the station in degrees |
| 19-27 | East longitude of the station in degrees |
| 29-36 | Altitude of the station in feet |
| 38-39<br>(41-42) | If this station reports P or $\dot{P}$ data (Q or $\dot{Q}$), these columns contain the two letter symbols for the associated station(s) of the tracking configuration. Each such associated station must appear on a separate Station Location card, but it is not necessary for columns 38 through 42 to be filled out on the latter. |

The last Station Location card must be followed by a card with the letters TR in columns 1 and 2. There may be up to 50 stations entered.

5.2.5.4 Data Specification

Load Sheet I

Column

| | |
|---|---|
| 1-2 | Station Call Letters<br>These must correspond to the letters on some Station Location card |
| 9-16 | Interval, in minutes, at which data for this station is to be generated |
| 18-23 | Minimum elevation at which the vehicle is visible |
| 25-30 | Maximum elevation at which the vehicle is visible<br>(Zero value will be set to $90^{\circ}$) |
| 32-40 | Maximum range (in n mi) to which vehicle is visible<br>(Zero value causes this test to be ignored) |
| 51-58 | Start time, from midnight of start date<br>(Zero value implies epoch is start time)<br>51-52 days<br>54-55 hours<br>57-58 minutes |
| 60-67 | Stop time, from midnight<br>60-61 days<br>63-64 hours<br>66-67 minutes |

The last card of this type must be followed by a card with the letters TR in columns 1 and 2.

Load Sheet II

Column

| | |
|---|---|
| 1-2 | Station Call Letters<br>These must correspond to the letters on some card from Load Sheet I. |
| 7-18 | An X in the appropriate column will cause the quantity listed above that column to be computed and used internally. |

The last card of this type must be followed by a card with the letters TR in columns 1 and 2.

The only limit on the number of cards using these formats is that at most fifty different stations are allowed.

Table 12. TRACE - Error Analysis Input Data

**X-1** **7090 INPUT DATA**

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PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ____ OF ____

| 1 | 7 | 73 |
|---|---|---|
| H1 | | |
| H2 | | |
| | | |
| | | |

| SYMBOL | PRE (1 19 37 55) | LOC. (2 20 38 56) | VALUE (7 25 43 61) | EXP. (17 35 53 71) |
|---|---|---|---|---|
| 1 | D | ITIN | | |
| 2 | I | YEAR | | |
| 3 | I | MNTH | | |
| 4 | I | DAY | | |
| 5 | | TZNE | | |
| 6 | | HR | | |
| 7 | | MIN | | |
| 8 | | SEC | | |
| 9 | I | ICTYP | | |
| 10 | | IC | | |
| 11 | | 2 | | |
| 12 | | 3 | | |
| 13 | | 4 | | |
| 14 | | 5 | | |
| 15 | | 6 | | |
| 16 | | DRAG | | |
| 17 | I | 2 | | |
| 18 | | 3 | | |
| 19 | | 4 | | |
| 20 | | 5 | | |
| | | | trajectory, residuals, partials, ATA, var. eq., elements, constants, update, z=0;β=90, obs.times | |
| 21 | D | PRCDE | | |
| 22 | I | PRTIM | | |
| 23 | I | 2 | | |
| 24 | | 3 | | |
| 25 | | 4 | | |
| 26 | | 5 | | |
| 27 | | 6 | | |
| 28 | | 7 | | |
| 29 | D | CPRAM | | |
| 30 | D | DPRAM | | |
| 31 | D | ΦPRAM | | |
| 32 | D | RPRAM | | |
| 33 | D | | | |
| 34 | D | | | |
| 35 | D | | | |
| 36 | D | | | |

| SYMBOL | PRE (1 19 37 55) | LOC. (2 20 38 56) | VALUE (7 25 43 61) | EXP (17 35 53 71) |
|---|---|---|---|---|
| 37 | | SIGMA | | |
| 38 | | 2 | | |
| 39 | | 3 | | |
| 40 | | | | |
| 41 | | | | |
| 42 | | | | |
| 43 | | | | |
| 44 | | | | |
| 45 | D | PRCΦV | | |
| 46 | D | ΦPBΦX | | |
| 47 | | | | |
| 48 | | | | |
| 49 | | CΦVQ | | |
| 50 | | | | |
| 51 | | | | |
| 52 | | | | |
| 53 | | | | |
| 54 | | | | |
| 55 | | | | |
| 56 | | | | |
| 57 | | | | |
| 58 | | | | |
| 59 | | | | |
| 60 | | | | |
| 61 | | IFLAG | | |
| 62 | I | 7 | | |
| 63 | I | 15 | | |
| 64 | | | | |
| 65 | | | | |
| 66 | | | | |
| 67 | | | | |
| 68 | | | | |
| 69 | | | | |
| 70 | | | | |
| 71 | | | | |
| 72 | | | | |
| 73 | | | | |
| 74 | | | | |
| 75 | | | | |
| 76 | | | | |

AEROSPACE FORM 2423 REV 6-62

# Table 13. Error Analysis Input Description

1: Itinerary. Specify order of computations.
1-Tracking data input, 2-Track,
3-Trajectory only
4-Data generation
5-Error analysis

2-8: Epoch

9: Type <u>1 2 3 4</u> of initial conditions.

10-15: IC

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| $x$ | $\alpha$ | $a$ | $\ell$ |
| $y$ | $\delta$ | $e$ | $\delta$ |
| $z$ | $\beta$ | $i$ | $\beta$ (ft-sec-deg) |
| $\dot{x}$ | $A$ | $\Omega$ | $A$ |
| $\dot{y}$ | $r$ | $\omega$ | $r$ |
| $\dot{z}$ | $v$ | $\tau$ | $v$ |

NOTE: If r is negative it is interpreted as height; if v is neg., circular velocity is computed and used.

16: $C_DA/W$. $(ft^2/lb)$

17: Atmosphere model specification
0 ARDC 59
1 Lockheed
2 Paetzold II
3 Paetzold I
4 L. F. E.

18: F - solar radio flux
$A_p$ - planetary magnetic index
g(a) - plasma intensity coeff.

21: X to print

| trajectory | residuals | partials | ATA Accum. | var. eq. | elements | constants | update | z=0;β=90 | obs. times |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

22-28: Print at $t=t_o(\Delta t_1)t_1(\Delta t_2)\ldots(\Delta t_n)t_n$ in minutes from midnight if PRTIM=1, from epoch if 0.
23 - $n(\leq 9)$, 24 - $t_o$, 25 - $\Delta t_1$
26 - $t_1$, 27 - $\Delta t_2$, 28 - $t_2$, etc.

29: Initial Condition Parameter Specification
$\boxed{P}$ Parameter to be solved for.
$\boxed{Q}$ Parameter considered in error.

| Type | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | $x$ | $y$ | $z$ | $\dot{x}$ | $\dot{y}$ | $\dot{z}$ | $t_o$ |
| 2 | $\alpha$ | $\delta$ | $\beta$ | $A$ | $r$ | $v$ | $t_o$ |
| 3 | $a$ | $e$ | $i$ | $\Omega$ | $\omega$ | $\tau$ | $t_o$ |

30: Differential Equation Parameter Specification
$\boxed{P}$ Parameter to be solved for.
$\boxed{Q}$ Parameter considered in error.

| $\frac{C_DA}{W}$ | $\mu$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_{21}^*$ | $J_{31}$ | $J_{41}$ | $J_{22}$ | $J_{32}$ | $J_{42}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |

31: Extension of Diff. Eq. Parameter Specification
$\boxed{P}$ Parameter to be solved for.
$\boxed{Q}$ Parameter considered in error

| $J_{33}$ | $J_{43}$ | $J_{44}$ | $L_{21}^*$ | $L_{31}$ | $L_{41}$ | $L_{22}$ | $L_{32}$ | $L_{42}$ | $L_{33}$ | $L_{43}$ | $L_{44}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |

32-36: Radar Parameter Specification.
$\boxed{P}$ to solve for
$\boxed{Q}$ parameter in error

| S | T | L | ℓ | A | R | A | E | $\dot{R}$ | T | u | v |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |

ST: Station Symbols — R: Range Bias
L: Station Latitude — A: Azimuth Bias
ℓ: Station Longitude — E: Elevation Bias
A: Station Altitude — $\dot{R}$: Range Rate Bias
T: Time Bias
u: "u" Bias
v: "v" Bias

37-44: Sigmas
$(R, A, E, \dot{R}, \dot{P}, \dot{Q}, P, Q, u, v, r)_I$
$I = 0, 1, 2, \ldots, 9$
I is the sigma type as referred to by the Station Location Data.

45: Covariance Matrix Output Specification.
$\boxed{X}$ in box for whole matrix
$\boxed{D}$ in box for sq. rt. of diagonals

| $C(P_o)$ | $C(X_{t1})$ | $C(E_{t1})$ | $C(R_{t1})$ | $C(a_{t1})$ | $C(T_{t1})$ | $C(P_2)$ | $C(X_{t2})$ | $C(E_{t2})$ | $C(R_{t2})$ | $C(a_{t2})$ | $C(T_{t2})$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |

46: Option Boxes: The form is:
OPBOX $\boxed{A}\boxed{B}\boxed{C}$
A. If "A" box contains an X the $\partial P/\partial Q$ will be printed.
B. If "B" box contains an X the $A^TA$ will not be printed.
C. If "C" box contains: 1, the $A^TA$ will be punched; 2, the partitioned $A_p{}^TA_p$ will be punched

49-60: C(Q) covariance matrix in lower triangular form.

62: Input control for station location and ephemeris cards.

63: $A^TA$ or $(A^TA)^{-1}$ input option.
-1, hold $A^TA$ from previous case
0, no $A^TA$ or inverse input
1, input $A^TA$ into ATA area
2, input inverse into ATA(501) area.

* or shorter list depending on ϕBJT

# Table 14. Radar Station Location Data

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Table 14. Radar Station Location Data

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ______ OF ______

| 1 3 | 5 6 | 9 10 11 12 13 14 15 16 17 | 19 20 21 22 23 24 25 26 27 | 29 30 31 32 33 34 35 36 | 38 39 | 41 42 | 73 74 75 76 77 78 79 80 |
|---|---|---|---|---|---|---|---|
| ST | TYPE | LAT (NORTH)° | LONG (EAST)° | ALTITUDE (FT) | $P\dot{P}$ ST | $Q\dot{Q}$ ST | IDENTIFICATION |

ST is the station identification code.

$P, \dot{P}$ ST must be input if either or both of the two quantities are selected on the "Ephemeris or Simulation Data Specification II" Load Sheet. (Likewise for $Q$, $\dot{Q}$ ST)

AEROSPACE FORM

## Table 15. Data Generation or Error Analysis Data Specification I

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Table 15. Data Generation or Error Analysis Data Specification I

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ______ OF ______

| 1 2 | 9 10 11 12 13 14 15 16 | 18 19 20 21 22 23 | 25 26 27 28 29 30 | 32 33 34 35 36 37 38 39 40 | 51 52 53 54 55 56 57 58 | 60 61 62 63 64 65 66 67 68 |
|---|---|---|---|---|---|---|
| ST | INTERVAL (MIN) * | MIN. EL (DEG) * | MAX. EL (DEG) * | MAX. RANGE (N. MILES) * | START TIME DA HR MN | STOP TIME DA HR MN |

ST is the station identification code.

START and STOP times are from midnight of epoch.

If start time is left zero, epoch will be defined as start time

An * following the unit value means a decimal point is necessary for that quantity.

AEROSPACE FORM

# Table 16. Error Analysis Data Specification II

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Table 16. Error Analysis Data Specification II

PROGRAMMER ________ KEYPUNCHED ________ VERIFIED ________ DATE ________ PAGE ____ OF ____

| 1–2 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18–80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ST | Range | Azimuth | Elevation | Range Rate | P Dot | Q Dot | P | Q | u | v | r | |

ST is the station identification code.

An X in a column will cause the appropriate quantity to be computed and used internally.

If P, P dot, Q, or Q dot are selected, the specification of an additional station identification code is necessary. (See Radar Location Data Input Sheet)

AEROSPACE FORM

5.2.6 Deck Setup

TRACE may be run from either binary cards or tape. This section describes the setup, from the user's point of view, of each mode, indicates how to produce a program tape, and explains the use of the dummy routines provided with the deck.

5.2.6.1 Running from Tape

At this point it is necessary to introduce REIN, a single program with only one function: to read CHAIN (the first link of TRACE) from some specified tape and thus initiate execution from this tape.

When a binary program tape is available, the input setup is as shown in Figure 16. REIN can be considered a loader, which calls CHAIN from logical unit 8 (A-8 with the present Aerospace Unit Table). REIN must be reassembled if any other unit is desired.

All comments below pertaining to the deck organization when running from cards are applicable to tape also, as every execution from cards first produces (and then uses) a program tape.

When using a previously written tape, it is not possible to compile and then use any program other than REIN.

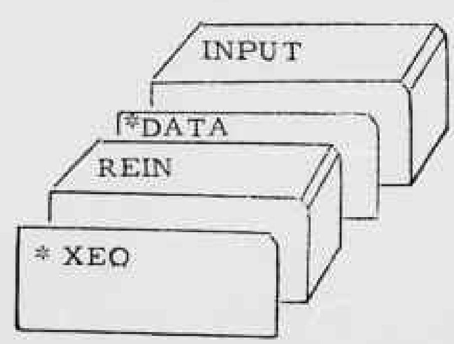


Figure 16. Running from Tape

5.2.6.2 Running from Cards

5.2.6.2.1 Arrangement of Deck

The complete program consists of five links: CHAIN, TRAIN, MAIN, GAIN, and FEIGN. (Use of REIN applies to execution from tape only.) A standard all-purpose deck would look like Figure 17.

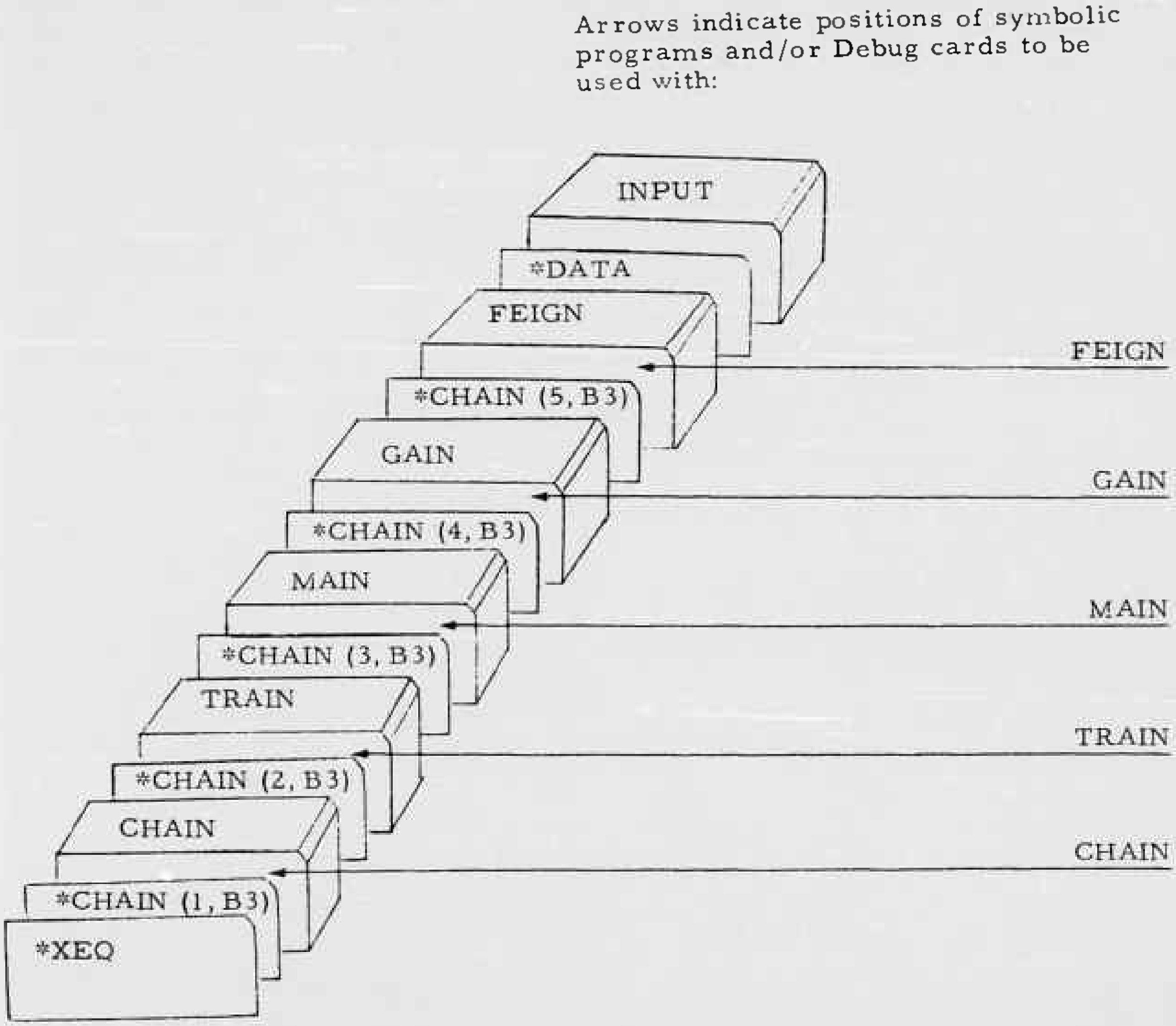


Figure 17. Running from Cards—Complete Deck

For various reasons, a variety of smaller decks or tapes may prove more desirable. It is necessary to include, for any given run, only those links to be executed during that run. The types of execution and the links that each require are:

| | |
|---|---|
| Tracking | CHAIN, TRAIN, MAIN |
| Trajectory only | CHAIN, MAIN |
| Data generation | CHAIN, GAIN |
| Error analysis | CHAIN, FEIGN |

Some economy in tape-handling results from the use of shorter decks in production work.

The input quantity, PTAPE = 11, must be included in the Basic Data when running from cards (see paragraph 5. 2. 6. 2. 2).

If symbolic cards for compilation are to be included, they must immediately follow the CHAIN control cards and precede the DEBUG cards (if any), for the appropriate link. In this connection, it is usually worthwhile, but not necessary, to remove the corresponding binary program from the link.

5. 2. 6. 2. 2 Producing a Tape

While it is true that every run from cards automatically produces a program tape, a short explanation of the CHAIN control card preceding each link may clarify the process.

The card

```
*          CHAIN (I, B3)          (I = 1, 2, 3, . . . )
```

assigns the number I to the link it precedes, and directs the FORTRAN monitor to store this program on tape B3.

Execution begins, after each program has been stored on the tape designated by its control card, by reading back in the (physical) first link of the deck; this must, therefore, always be CHAIN. If the tape number (in this case, B3) is the same on each control card, a binary tape of TRACE is then available by simply saving B3. When source programs are included with the binary cards, the result of the compilation appears on the tape.

Links are brought into core by programming in TRACE, which assumes them to be on the tape designated by the input quantity PTAPE. PTAPE is set, in REIN, to 8, and does not have to be read during a normal run from tape. When using cards, however, PTAPE must be input. It must equal 11 unless the control cards are changed (B2 and A4 are the only other units FORTRAN will recognize for this purpose at present); or a unit table is employed in which B3 does not equal 11.

5.2.6.2.3 Dummy Routines

For some purposes, it may be desirable to increase the amount of core storage available for data handling.

Since TRACE contains many options, not all of which are usually executed in any one run, there are always a certain number of extraneous subroutines present. (For instance, only one integration routine is ever in use during any trajectory.) These routines may be replaced with one-word "dummies" if more storage cells are needed. (In replacing cards, it should be noted that the TRACE programs, and then the library routines, are alphabetical within each link.)

A list of a few subroutines of appreciable size that might be dummied are given here as an example.

| Link | Routine | When It Is Unnecessary | Approx. No. of Words |
|---|---|---|---|
| MAIN | SDUMP | No dump required | 150 |
| | DXDA | No analytic partials | 400 |
| | DXDRSR | DXDA being used | 350 |
| | PATTY | No accumulated normal matrix output | 400 |
| | PTRAJ | No trajectory output | 170 |
| | REST | BLIST (constraints) not used | 260 |
| | REST1 | " " " " | 215 |
| | AMRK | Gauss-Jackson integration used (COW) | 390 |
| | COW | AMRK used | 1100 |

5.2.6.3 Data Deck Setup

All types of runs require input of BASIC DATA followed by the END BASIC card. ITIN determines what further input is required.

| If ITIN = | 1 | 4 | 5 |
|---|---|---|---|
| Read | (TRAIN) Tracking Data Input | (GAIN) Data Generation | (FEIGN) Error Analysis |
| Station Location | 1st | 2nd | 2nd |
| Radar Observation | 2nd | | |
| Data Specification I and II | | 3rd | 3rd |
| END DATA Card | | 1st | 1st |

ITIN = 2, 3, requires only an END DATA card.

There is one exception to the above chart. If two or more GAIN or FEIGN runs are run in the same ITIN sequence, the Station Location and Data Specification cards are normally read the first time only. (See pages 5-28 and 5-43.)

Figure 18 shows an input deck for a single trajectory.

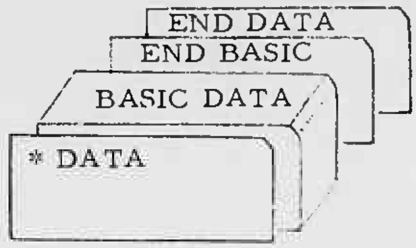


Figure 18. ITIN = 3, Trajectory Only

For two successive trajectories the deck in Figure 18 should be followed by the one in Figure 19.

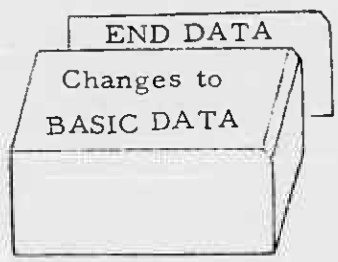


Figure 19. ITIN = 33, Two Trajectories

The input deck for a standard tracking run with observation cards is shown in Figure 20.

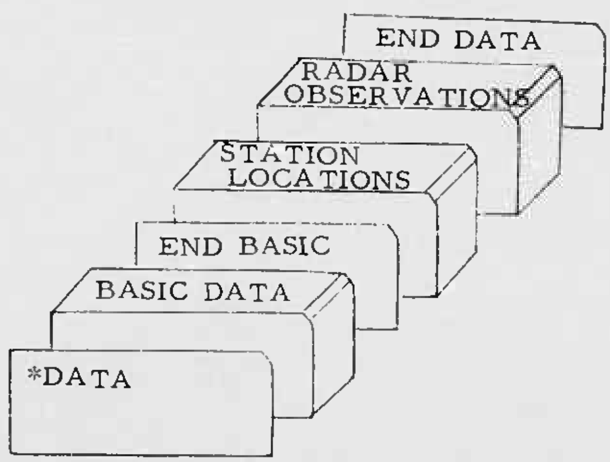


Figure 20. ITIN = 12, Tracking Input and Run

For a tracking run followed by data generation, the above deck should be followed by Figure 21.

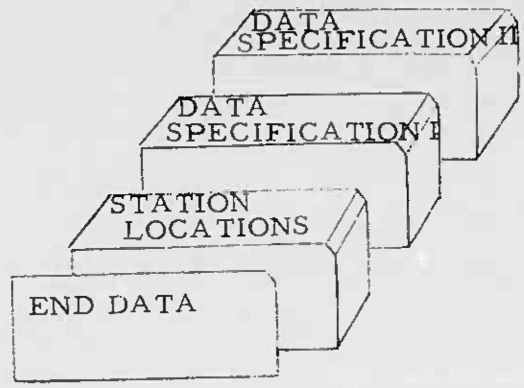


Figure 21. ITIN = 124, Tracking Run Plus Data Generation

Data generations and error analyses are run from input decks of identical structure, as in Figure 22.

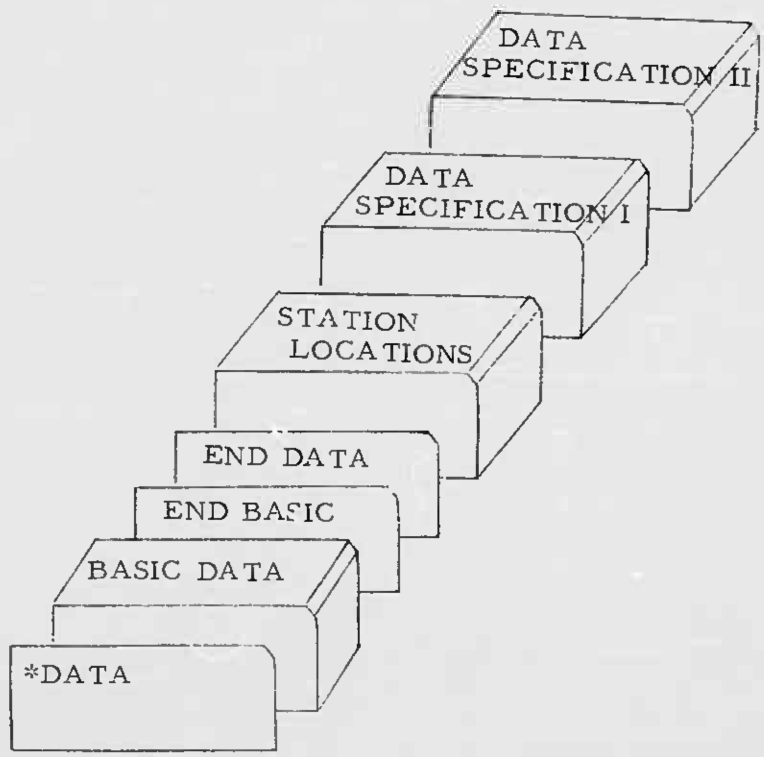


Figure 22. ITIN = 4 or 5, Data Generation or Error Analysis

## 5.3 OUTPUT

Each link, or type of computation, chosen in TRACE provides two types of programmed output. First, there are those headers and quantities that are a function only of the link being executed and are not controlled by input options. Second, there is output which must be specifically selected through the input quantities PRCDE, PRTIM, and some of the tape parameters. A third possible type of output is available, to anyone sufficiently familiar with the program, by use of the FORTRAN DEBUG capabilities.

The first two kinds of output can best be described by means of examples. Fifteen sample printouts are included at the end of this section.

### 5.3.1 Examples 1 and 2 (CHAIN)

The output from CHAIN is all of the first type and appears on every run.

The first 26 lines are the BCD card images of the FINP input. Inclusion of a card with the symbol CLOCK in columns 7 through 11 will cause the time (in hundredths of a minute), at which this card is read, to be output. Printing of the input cards may be eliminated by removing the special versions of subroutine (CSH)S from the binary deck. Any error printout indicates that the last card was either punched incorrectly, includes a symbolic location not in the FINP list, or is not a Basic Data card.

### 5.3.2 Example 3 (TRAIN)

TRAIN output is not input-controlled and is produced whenever a tracking run is executed. Example 3 is the result of specifying station location and radar observations on cards, along with the normal input deck. If a binary radar data tape had been read, TRAIN output would not include the observational data.

There are several possible error messages. In Example 3, a mispunched card, redundancy in reading the BCD tape, and inclusion of an observation from a station for which there is no station location card, would all produce the same effect. An appropriate line of output is printed, the observation (or station) in question is deleted, and execution continued. Redundancies in reading the binary data tape would cause this information to be printed and execution terminated.

5.3.3 Examples 4, 5, 6, and 7 (MAIN - Trajectory Only)

All output from MAIN during a trajectory run, with the exception of the initial conditions, must be selected through the input quantities PRCDE and PRTIM.

Example 4 results from the request that the constants in use in the program be printed out.

Example 5 shows trajectory output; Example 6, the variational equations; and Example 7, the elements. In these examples, the three kinds of output are shown singly; any two, or all three, could have been given at the same time just as easily. However, it is not possible to request one type of output at one sequence of times and another type at another sequence during the same trajectory.

5.3.4 Examples 8, 9, 10, and 11 (MAIN - Tracking)

When using TRACE as a tracking program, MAIN produces both input-independent and input-controlled output.

The first type consists of the initial conditions, and, for each iteration, something similar to Example 8. The legend, CURRENT SOLUTION IS NOT GOOD, indicates that the previous solution has caused the rms of the residuals to increase. The program therefore will decrease the bounds, return to the last good solution, and re-solve, using the corresponding normal matrix. CURRENT SOLUTION IS BEST SO FAR is a signal that the rms has decreased and that the bounds will be increased for faster convergence. SIGMA (PARAMETERS)/SIGMA (NORMALIZED DATA) is the square root of the diagonal of the inverse normal matrix.

The input-controlled output includes everything that can be obtained during a trajectory-only run (see paragraph 5.3.3), plus four additional computations.

Example 9 shows the STATION-BY-STATION SOS (square root of the sum of the squares of the residuals after division by the appropriate radar sigmas). This is printed once per iteration and is limited, at present, to nine stations.

Example 10 gives the measured-minus-computed values of the radar residuals. If a time bias is included from one or more stations, the output time will be biased.

The partials of the radar observations, with respect to the parameters being solved for, appear as in Example 11. This may be requested along with, or independently of, the residuals (Example 10). The units of the partials are in earth-radii and radians; (all other output is in feet and degrees, or any other system specified using the non-standard input); these quantities are output before the division by the radar sigmas.

### 5.3.5 Examples 12, 13, 14, and 15 (GAIN - Data Generation)

All the output illustrated in paragraph 5.3.3 is also available during a data generation. Besides this, there are three additional types of output:

Example 12 shows the station locations and data specifications.

Example 13 is a result of choosing to employ GAIN in the Rise-Set-Only mode.

Example 14 shows the various types of data that the program can produce.

### 5.3.6 Example 15 (FEIGN - Error Analysis)

All the output illustrated in paragraph 5.3.3 is also available for an error analysis (FEIGN) run. Example 15 illustrates the type of output obtainable by input control. Any combination of the matrices may be selected as output by option. Only the square roots of the diagonals of the covariance matrices may be specified as output, if desired. Printout occurs at the times specified by PRTIM.

5.3.7 Other Output

There are three additional means of acquiring output, but since each requires more familiarity with the programming of TRACE, they will only be mentioned here.

First, the FORTRAN DEBUG system may be used. All binary routines compiled from FORTRAN source decks are preceded by their symbol table, and almost all information of interest can be dumped from COMMON.

Second, the source programs themselves may be modified and recompiled.

Third, core dumps may be obtained in case of trouble or at the end of a run. To this end, each link contains, as its first program, a copy of SDUMP. A suitable manual transfer (to a location dependent on the version of FORTRAN in present use) will automatically produce an octal dump.

# EXAMPLE 1

```
IPTAPE11                                                                         FINP INPUT CARO   1
GINTEG   PARAMETERS OF THE TRAJECTORY INTEGRATION                                FINP INPUT CARO   2
11    1          12    2                                                         FINP INPUT CARO   3
 4    1           5    1             11   1.00002516                             FINP INPUT CARO   4
 22   .1         23    .1         -9 24   .5         -7 25   .2          -8      FINP INPUT CARO   5
 26   1                                                                          FINP INPUT CARO   6
 30   1          31    0             32   4             33   1           -7      FINP INPUT CARO   7
134   4         135    1             37   .001          38   .001                FINP INPUT CARO   8
GC    CONSTANTS.                                                                 FINP INPUT CARO   9
 1    .43752691 -2 2   .55303935 -2 6    .5                                      FINP INPUT CARO  10
 14   57.2957795  15   20925738.     16   332951.3      17   .0122999            FINP INPUT CARO  11
 18   .814979     19   .107821       20   317.887       21   95.129              FINP INPUT CARO  12
 22   23454.865   23   3443.9336     24   20925738.     25   348762.3            FINP INPUT CARO  13
 30   348762.3    31   1.5           32   1.0471976     33   3.14159265          FINP INPUT CARO  14
 34   298.3       36   82505.922     40   2.            41   1.                  FINP INPUT CARO  15
GOBJZ    EARTH MODEL - VALUES CONVERTEO FROM C(8)-C(12)                          FINP INPUT CARD  16
 1    4           2    .0010823      3    -.0000023     4    -.0000018           FINP INPUT CARO  17
H1    TEST CASE FOR PROGRAM TAPES                                                FINP INPUT CARO  18
H2    TRAJECTORY LINK                                                            FINP INPUT CARO  19
OITIN 3445       IYEAR 1962         IMNTH 6             IDAY  20                 FINP INPUT CARO  20
IICTYP4           IC   240.          2    15.           3    90.                 FINP INPUT CARO  21
 4    90.         5    21606900.     6    25460.        ORAG .015151515          FINP INPUT CARO  22
OPRCOEX   X X X  IPRTIMO            12    1             3    0                   FINP INPUT CARO  23
 4    5.          5    30.          OCPRAM2XXXXXX       IIBCOI7                  FINP INPUT CARO  24
IETAPE7                                                                          FINP INPUT CARO  25
ENO BASIC                                                                        FINP INPUT CARO  26
```

## EXAMPLE 2

```
                                                    +++++
                                               ++++       ++++
                                            +++               +++
TTTTTTTTTTT    RRRRRRRRR               +                  AA    +      CCCCCCCCCCC    EEEEEEEEEEE
TTTTTTTTTTT    RRRRRRRRRR             +                 AAAA     +     CCCCCCCCCCC    EEEEEEEEEEE
    TT         RR       RR            +               AA  AA     +     CC             EE
    TT         RRRRRRRRRR            +              AA    AA      +    CC             EEEEEE
    TT         RRRRRRRRR             +            AAAAAAAAAA      +    CC             EEEEEE
    TT         RR     RR             +          AAAAAAAAAAAA      +    CC             EE
    TT         RR      RR             +   AA              AA     +     CCCCCCCCCCC    EEEEEEEEEEE
    TT         RR        RR           + AA                AA     +     CCCCCCCCCCC    EEEEEEEEEEE
                                       +                        +
                                        +++                  +++
                                           ++++          ++++     (KEENER THAN MOST PROGRAMS)
                                                 +++++
```

TEST CASE FOR PROGRAM TAPES                                    TRAJECTORY LINK

| EPOCH | YEAR | MONTH | DAY | TZONE | HOUR | MIN | SEC |
|---|---|---|---|---|---|---|---|
| | 1962. | 6. | 20. | 0. | 0. | 0. | 0. |

EXAMPLE 2 (continued)

```
  END DATA                                                                              FINP INPUT CARD  27

TEST CASE FOR PROGRAM TAPES                                                  TRAJECTORY LINK

                                              INITIAL CONDITIONS

X    =      -0.17646601E 08            ALPHA=       0.14772789E 03            A=       0.21498925E 08
Y    =       0.11143698E 08            DELTA=       0.14999999E 02            E=       0.50222881E-02
Z    =       0.55922771E 07            BETA =       0.89999999E 02            I=       0.15000000E 02
XDOT=       -0.13594133E 05            AZ   =       0.89999999E 02            O=       0.57727896E 02
YDOT=       -0.21526986E 05            R    =       0.21606900E 08            U=       0.27000017E 03
ZDOT=        0.49731862E-03            V    =       0.25459999E 05            T=      -0.43992169E 02

  ATMOSPHERE - ARDC 1959        CDA/W = 0.01515     W/CDA =      66.

                                N1=4.          EARTH MODEL          N2=0.
GM=    0.553039E-02 J2=E        0.10823CE-02  J3=    -0.230000E-05  J4=E    -0.180000E-05  J5=E     0.
    J31=     0.                 L31=     0.                 J41=     0.                 L41=     0.
    J22=     0.                 L22=     0.                 J32=     0.                 L32=     0.
    J42=     0.                 L42=     0.                 J33=     0.                 L33=     0.
    J43=     0.                 L43=     0.                 J44=     0.                 L44=     0.

FORMULATION
            COWELL (EQS.OF MOTION)

DIFFERENTIAL EQUATION SUBROUTINE
            GAUSS-JACKSON        E BAR=  0.1000E-09      A= 1.000    RATIO OF COWELL STEP TO RUNGE KUTTA  4
STEP SIZE -     INITIAL=    0.1000E 01     MINIMUM=     0.              MAXIMUM=    0.4000E 01
 DO NOT RECOMPUTE PERTURBATIONS FOR CORRECTOR
```

## EXAMPLE 3

TRACKING OATA INPUT

STATIONS

| | SIG | REF | LATITUDE | LONGITUOE | HEIGHT |
|---|---|---|---|---|---|
| AA | 0. | 0. | 0.14500000E 02 | 0.26000000E 03 | 0. |
| BB | 0. | 0. | 0.13000000E 02 | 0.27999999E 03 | 0. |

OBSERVATIONS

| MO. | OAY | MIN. | ST TYPE | | | |
|---|---|---|---|---|---|---|
| 6 | 20 | 3.0000 | AA0001 | 0.30164570E 07 | 0.27280498E 03 | 0.91987299E 01 |
| 6 | 20 | 4.0000 | AA0001 | 0.16738790E 07 | 0.27024341E 03 | 0.21941149E 02 |
| 6 | 20 | 5.0000 | AA0001 | 0.70079199E 06 | 0.22254068E 03 | 0.74764339E 02 |
| 6 | 20 | 6.0000 | AA0001 | 0.14761499E 07 | 0.10213792E 03 | 0.25379589E 02 |
| 6 | 20 | 7.0000 | AA0001 | 0.28036570E 07 | 0.98899499E 02 | 0.10195000E 02 |
| 6 | 20 | 8.0000 | BB0001 | 0.32057820E 07 | 0.26889464E 03 | 0.78120099E 01 |
| 6 | 20 | 9.0000 | BB0001 | 0.18938670E 07 | 0.25949971E 03 | 0.17961360E 02 |
| 6 | 20 | 10.0000 | BB0001 | 0.94618399E 06 | 0.21110356E 03 | 0.42586059E 02 |
| 6 | 20 | 11.0000 | BB0001 | 0.14894509E 07 | 0.12938703E 03 | 0.23851609E 02 |
| 6 | 20 | 12.0000 | BB0001 | 0.27485510E 07 | 0.11468191E 03 | 0.98695399E 01 |

PARAMETERS TO BE CORRECTEO

ALPHA OELTA BETA A R V

10 OBSERVATION TIMES, 30 OBSERVATIONS, 2 STATIONS, 50 CELLS IN COMPACTEO OATA LIST
1 FLOCKS (IF BCD INPUT)

3 ITERATIONS (MAXIMUM), 6 PARAMETERS

## EXAMPLE 4

CONSTANTS

| | DECIMAL | OCTAL | | DECIMAL | OCTAL |
|---|---|---|---|---|---|
| OMEGA E | 0.43752690E-02 | 171436571527 | GM | 0.55303935E-02 | 171552341155 |
| ALPHA G | 0.46727332E 01 | 203453034077 | 1/EPS. | 0.29830000E 03 | 211452231463 |
| EARTH RADIUS - FT | 0.20925738E 08 | 231477232250 | REL MASS-SUN | 0.33295130E 06 | 223505113515 |
| REL MASS-MOON | 0.12299900E-01 | 172623026050 | REL MASS-VENUS | 0.81497900E 00 | 200641211666 |
| REL MASS-MARS | 0.10782100E-00 | 175671505015 | REL MASS-JUPITER | 0.31788700E 03 | 211475706112 |
| REL MASS-SATURN | 0.95129000E 02 | 207574410142 | E.R./A.U. | 0.23454864E 05 | 217556366727 |
| N.M./E.R. | 0.34439336E 04 | 214656373600 | 1-D, DISTANCE | 0.20925738E 08 | 231477232250 |
| I-D, VELOCITY | 0.34876230E 06 | 223524455115 | DELTA F FACTOR | 0. | 000000000000 |
| DELTA T FACTOR | 0. | 000000000000 | FT/SEC//E.R./MIN | 0.34876230E 06 | 223524455115 |
| DEG/SEC//RAD/MIN | 0.10471976E 01 | 201414052221 | | | |

## EXAMPLE 5

| 6/20/62<br>ME,MM,ST,DT | X,R | XDOT,V | LAT,LONG,H,SBV | ALPHA,DELTA,BETA,A | REV | |
|---|---|---|---|---|---|---|
| 0. | -0.17646601E 08 | -0.13594133E 05 | 15.09648 | 147.72789 | | 0 |
| 0. | 0.11143698E 08 | -0.21526986E 05 | 240.00000 | 15.00000 | | |
| 0. | 0.55922771E 07 | 0.49731862E-03 | 112.88182 | 90.00000 | | |
| 0.250 | 0.21606899E 08 | 0.25459999E 05 | 15.09340 | 90.00000 | | |

| 6/20/62<br>ME,MM,ST,DT | X,R | XDOT,V | LAT,LONG,H,SBV | ALPHA,DELTA,BETA,A | REV | |
|---|---|---|---|---|---|---|
| 5.000 | -0.20541991E 08 | -0.55040607E 04 | 14.14049 | 168.64070 | | 0 |
| 5.000 | 0.41268076E 07 | -0.24761589E 05 | 259.65939 | 14.04960 | | |
| 300.000 | 0.52432942E 07 | -0.23025117E 04 | 111.40926 | 90.12443 | | |
| 0.250 | 0.21598519E 08 | 0.25470228E 05 | 14.13762 | 95.31576 | | |

## EXAMPLE 6

PARTIALS OF X,Y,....,ZDOT WITH RESPECT TO

| ALPHA | DELTA | A | R | DRAG |
|---|---|---|---|---|
| 1.8340E 05 | 3.9334E 04 | -7.1665E 04 | -1.9681E 00 | 6.1295E 02 |
| -3.2471E 05 | -2.5143E 04 | 4.5380E 04 | 5.0342E-01 | 2.0963E 03 |
| -1.4569E-02 | 1.7628E 05 | -3.1711E 05 | 5.1928E-01 | 1.7570E 02 |
| 3.8301E 02 | -8.6900E 01 | -4.6400E 01 | -2.6592E-03 | 1.0158E 00 |
| 2.0444E 02 | 5.3920E 01 | 2.9753E 01 | -8.1933E-04 | 5.0649E 00 |
| -5.7837E-05 | -3.7797E 02 | -2.0688E 02 | 4.8729E-04 | 5.4496E-01 |

## EXAMPLE 7

| A,E,I,O,U,T | | | | |
|---|---|---|---|---|
| 0.21502092E 08 | MEAN ANM= | 312.23124 | APOGEE= | 3564.28851 |
| 0.72037587E-02 | TRUE ANM= | -48.38369 | HT = | 121.11040 |
| 0.15015820E 02 | ODOT= | -5.83267 | PERIGEE= | 3513.30322 |
| 0.57584282E 02 | UDOT= | 11.06434 | HT = | 70.12511 |
| 0.26097973E 03 | | | PERIOD(K)= | 88.00389 |
| -0.46326566E 02 | | | PERIOD(A)= | 87.91772 |
| | | | PERIOD(N)= | 87.75429 |

## EXAMPLE 8

```
ITERATION      1
   30 OATA POINTS WERE USEO IN THE SOLUTION,     0 OATA POINTS WERE EOITEO
CURRENT SOLUTION IS BEST SO FAR

CURRENT SOLUTION IS

     ALPHA              OELTA              BETA               A                  R                  V
   0.14772789E 03    0.14999999E 02    0.89999999E 02    0.89999999E 02    0.21606900E 08    0.25459999E 05

SOLUTION IN MACHINE UNITS (OCTAL)
     202512015757      177414052215      201622077324      201622077324      201410252447      175453006154

RMS=    0.35569025E-01   FOR THIS SOLUTION

CORRECTIONS
   0.18391670E-04   -0.44577881E-05    0.78051671E-05   -0.50069900E-05    0.96433450E 01   -0.52227319E-02

BOUNOS
   0.99999999E 00    0.99999999E 00    0.99999999E 00    0.99999999E 00    0.99999999E 05    0.99999999E 03

NEXT SOLUTION IS
   0.14772791E 03    0.14999995E 02    0.90000007E 02    0.89999993E 02    0.21606909E 08    0.25459994E 05

SOLUTION IN MACHINE UNITS (OCTAL)
     202512015771      177414052170      201622077335      201622077316      201410252505      175453006133

PREOICTED RMS=    0.33840312E-01   FOR NEXT SOLUTION


SIGMA(PARAMETERS)/SIGMA(NORMALIZEO DATA)

       1                  2                  3                  4                  5                  6

     ALPHA              OELTA              BETA               A                  R                  V
   0.86825364E-03    0.92230076E-03    0.24524249E-02    0.16469835E-02    0.47865885E 03    0.64891564E 00

CORRELATION MATRIX

               1          2          3          4          5          6
  1          1.000
  2         -0.042      1.000
  3          0.031     -0.124      1.000
  4          0.109      0.896     -0.119      1.000
  5          0.339     -0.125      0.924     -0.061      1.000
  6         -0.917      0.039     -0.079     -0.093     -0.353      1.000
```

## EXAMPLE 9

TRACKING LINK

RESIDUALS

| ST | TIME | R | A | E | R DOT | P DOT | Q DOT | P | Q |
|---|---|---|---|---|---|---|---|---|---|
| AA | 3.000 | -4.5564E 01 | 1.3660E-05 | -1.5591E-03 | 0. | 0. | 0. | 0. | 0. |
| AA | 4.000 | -6.5053E 01 | 8.1962E-05 | -2.4879E-03 | 0. | 0. | 0. | 0. | 0. |
| AA | 5.000 | -9.7482E 01 | -8.1484E-03 | -4.0298E-04 | 0. | 0. | 0. | 0. | 0. |
| AA | 6.000 | -1.2668E 01 | -2.1088E-04 | -4.0768E-03 | 0. | 0. | 0. | 0. | 0. |
| AA | 7.000 | 1.4616E 01 | -6.4033E-05 | -2.2672E-03 | 0. | 0. | 0. | 0. | 0. |
| BB | 8.000 | -8.2866E 01 | 1.9808E-04 | -1.5147E-03 | 0. | 0. | 0. | 0. | 0. |
| BB | 9.000 | -9.7989E 01 | -7.2400E-04 | -2.2956E-03 | 0. | 0. | 0. | 0. | 0. |
| BB | 10.000 | -9.7998E 01 | -5.6759E-03 | -3.7101E-03 | 0. | 0. | 0. | 0. | 0. |
| BB | 11.000 | 1.1362E 01 | -1.7315E-03 | -5.2563E-03 | 0. | 0. | 0. | 0. | 0. |
| BB | 12.000 | 5.1528E 01 | -5.1226E-04 | -2.9673E-03 | 0. | 0. | 0. | 0. | 0. |

## EXAMPLE 10

SOS FOR AA

| TYPE OF OBS. | R | A | E |
|---|---|---|---|
| NO. OF OBS. | 5 | 5 | 5 |
| SOS | 0.1272E 03 | 0.8152E-02 | 0.5527E-02 |
| RMS | 0.5689E 02 | 0.3646E-02 | 0.2472E-02 |
| RMS/SIG | 0.5689E 00 | 0.3646E-01 | 0.2472E-01 |

SOS FOR BB

| TYPE OF OBS. | R | A | E |
|---|---|---|---|
| NO. OF OBS. | 5 | 5 | 5 |
| SOS | 0.1699E 03 | 0.6003E-02 | 0.7600E-02 |
| RMS | 0.7597E 02 | 0.2685E-02 | 0.3399E-02 |
| RMS/SIG | 0.7597E 00 | 0.2685E-01 | 0.3399E-01 |

## EXAMPLE 11

```
                                          OATA GENERATION LINK


STATIONS
       SIG   REF          LATITUOE         LONGITUOE            HEIGHT

AA     -0.   -0.         14.50000000      260.00000000            0.
88     -0.   -0.         13.00000000      280.00000000            0.

DATA
         INTERVAL    MIN ELEV   MAX ELEV   MAX RANGE      START         STDP
            MIN        OEG        OEG        N MI      DA HR MIN     OA HR MIN
AA        1.0000      5.000      -0.          -0.      -0 -0 -0      0  0 30
88        1.0000      5.000      -0.          -0.      -0 -0 -0      0  0 30

OATA TYPES                                             R
                     R    P    Q              A    E  OBL  MUT            SUR       DOP ASP OBS
        R    A    E OOT  DOT  DDT   P    Q   OOT  DOT OOT  VIS  LAT  LNG  RNG   H   RAT ANG VAR
AA      X    X    X
88      X    X    X
```

## EXAMPLE 12

```
PARTIALS FOR AA       3.000(R)
 -9.5505207E-01  -4.1489478E-02    8.9398626E-03    5.0486815E-01    3.7217408E-06   -4.8568727E-02

  0.
```

## EXAMPLE 13

```
AA                                JUNE  20,  1962

    RISE (   5.00 DEGREES ELEV.)      0. HOURS    2.48 MINUTES    AZIMUTH 273.481 DEGREES

    SET  (   5.00 OEGREES ELEV.)      0. HOURS    7.70 MINUTES    AZIMUTH  98.094 DEGREES


BB                                JUNE  20,  1962

    RISE (   5.00 OEGREES ELEV.)      0. HOURS    7.61 MINUTES    AZIMUTH 270.952 OEGREES

    SET  (   5.00 DEGREES ELEV.)      0. HOURS   12.66 MINUTES    AZIMUTH 110.960 DEGREES
```

# EXAMPLE 14

AA JUNE 20, 1962

| HRS | MINS | T-ST | RANGE NAUT MILES | AZIMUTH OEGREES | ELEVATION OEGREES | RANGE RATE FT/SEC | P DOT FT/SEC | Q DOT FT/SEC | P NAUT MILES | Q NAUT MILES |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | AZMTH RATE OEG/MIN | ELEV RATE DEG/MIN | R DBL DOT FT/SEC**2 | MUTUAL VIS | LAT DEGREES | LONG DEGREES | SURF RANGE NAUT MILE | HEIGHT NAUT MILE |
| | | | ODP RATE | LOOK ANGLE OEGREES | VARIANCES R | A | E | ROOT | ADOT | EDOT |

RISE ( 5.00 DEGREES ELEV.) 0. HOURS 2.48 MINUTES AZIMUTH 273.481 OEGREES

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 3.00 | 3.00 | 496.447 | 272.805 | 9.220 | -22879.865 | -9279.470 | -9279.470 | -5009.279 | -5009.279 |
| | | | -1.306 | 7.934 | 5.255 | 10000.000 | 14.750 | 251.817 | 476.353 | 112.348 |
| | | | -0. | 90.000 | 0. | 0. | 0. | 0. | 0. | 0. |
| 0. | 4.00 | 4.00 | 275.486 | 270.243 | 21.954 | -21444.488 | -7288.964 | -7288.964 | -5093.182 | -5093.182 |
| | | | -4.973 | 21.389 | 51.749 | 10000.000 | 14.482 | 255.743 | 247.831 | 111.936 |
| | | | -0. | 90.000 | 0. | 0. | 0. | 0. | 0. | 0. |
| 0. | 5.00 | 5.00 | 115.336 | 222.541 | 74.759 | -3710.331 | 10980.196 | 10980.196 | -5110.891 | -5110.891 |
| | | | -357.854 | 67.985 | 765.919 | 10000.000 | 14.140 | 259.659 | 29.254 | 111.410 |
| | | | -0. | 90.000 | 0. | 0. | 0. | 0. | 0. | 0. |
| 0. | 6.00 | 6.00 | 242.944 | 102.138 | 25.391 | 20814.283 | 36018.412 | 36018.412 | -4835.664 | -4835.664 |
| | | | -6.765 | -26.926 | 80.052 | 10000.000 | 13.727 | 263.563 | 212.864 | 110.773 |
| | | | 0. | 90.000 | 0. | 0. | 0. | 0. | 0. | 0. |
| 0. | 7.00 | 7.00 | 461.424 | 98.899 | 10.215 | 22804.189 | 38499.121 | 38499.121 | -4464.604 | -4464.604 |
| | | | -1.535 | -8.887 | 10.379 | 10000.000 | 13.244 | 267.453 | 441.480 | 110.029 |
| | | | 0. | 90.000 | 0. | 0. | 0. | 0. | 0. | 0. |

SET ( 5.00 OEGREES ELEV.) 0. HOURS 7.70 MINUTES AZIMUTH 98.094 OEGREES

EXAMPLE 15

```
                                   ERROR ANALYSIS LINK

STATIONS
       SIG   REF        LATITUDE        LONGITUOE            HEIGHT

AA     -0.   -0.       14.50000000     260.00000000            0.
BB     -0.   -0.       13.00000000     280.00000000            0.

OATA
        INTERVAL    MIN ELEV    MAX ELEV    MAX RANGE        START            STOP
           MIN         OEG         OEG        N MI        OA HR MIN       OA HR MIN
AA       1.0000       5.000       -0.           -0.       -0 -0 -0         0  0 30
BB       1.0000       5.000       -0.           -0.       -0 -0 -0         0  0 30

DATA TYPES
                      R    P    Q                      LIL
      R    A    E   OOT  DOT  OOT   P    Q    U    V    R
AA    X    X    X
BB    X    X    X

PARAMETERS TO BE CORRECTEO
ALPHA  OELTA   BETA   A      R      V      LAT.   LAT.
                                          AA  00 BB  00

PARAMETER ERRCRS
 LONG   RBIAS  LONG   RBIAS
AA  00 AA  00 BB  00 BB  00
```

EXAMPLE 15 (continued)

ATA AT 0.13000000E 02 MIN.

| ALPHA | DELTA | A | R | DRAG |
|---|---|---|---|---|
| 8.6935E 07 | 3.2838E 06 | -6.1192E 05 | -1.5440E 02 | -1.3128E 05 |
| | 7.5625E 06 | -6.1403E 06 | -4.2227E 01 | 1.6025E 04 |
| | | 5.2609E 06 | 2.7122E 01 | -1.8073E 04 |
| | | | 5.5390E-04 | 1.8449E-01 |
| | | | | 3.5411E 02 |

ATA INVERSE

| | | | | |
|---|---|---|---|---|
| 6.0075E-08 | | | | |
| 4.6899E-07 | 1.5113E-05 | | | |
| 4.4288E-07 | 1.3810E-05 | 1.3326E-05 | | |
| 2.7752E-02 | 6.5565E-01 | 5.4530E-01 | 3.6312E 04 | |
| 9.1931E-06 | -1.4685E-04 | -6.4719E-05 | -1.0470E 01 | 1.5030E-02 |

SQUARE ROOT OF DIAGONAL OF INVERSE

| | | | | |
|---|---|---|---|---|
| 2.4510E-04 | 3.8876E-03 | 3.6505E-03 | 1.9056E 02 | 1.2260E-01 |

INVERSE WITH Q PARAMETER EFFECT

| | | | | |
|---|---|---|---|---|
| 5.3561E-05 | | | | |
| -6.4694E-04 | 7.8494E-03 | | | |
| 3.1092E-04 | -3.7433E-03 | 1.8151E-03 | | |
| -4.5100E 00 | 5.5567E 01 | -2.5789E 01 | 4.2119E 05 | |
| -2.6792E-02 | 3.2418E-01 | -1.5560E-01 | 2.2627E 03 | 1.3441E 01 |

SQUARE ROOT OF DIAGONAL

| | | | | |
|---|---|---|---|---|
| 7.3185E-03 | 8.8597E-02 | 4.2604E-02 | 6.4899E 02 | 3.6662E 00 |

EXAMPLE 15 (continued)

ORBIT PLANE COVARIANCE MATRIX 1

| XI | ETA | ZETA | XIDOT | ETADOT | ZETADOT |
|---|---|---|---|---|---|
| 1.6869E 05 | | | | | |
| -3.2160E 04 | 3.3869E 04 | | | | |
| -8.0532E 04 | 7.0253E 04 | 2.7522E 05 | | | |
| 3.1395E 02 | -4.1832E 01 | -1.2072E 02 | 6.0283E-01 | | |
| 1.1310E 02 | 3.6433E 01 | 5.3523E 01 | 2.5594E-01 | 2.0594E-01 | |
| -6.8012E 02 | 3.2630E 02 | 1.0666E 03 | -1.1548E 00 | -6.3429E-02 | 5.2173E 00 |
| SQUARE ROOT OF DIAGONAL | | | | | |
| 4.1072E 02 | 1.8404E 02 | 5.2461E 02 | 7.7642E-01 | 4.5380E-01 | 2.2841E 00 |

| VOLUME 1 | VOLUME 2 |
|---|---|
| 0.24483448E 08 | 0.25768173E-00 |

POLAR COVARIANCE MATRIX 1

| ALPHA | DELTA | BETA | A | R | V |
|---|---|---|---|---|---|
| SQUARE ROOT OF DIAGONAL | | | | | |
| 7.4756E-04 | 1.2796E-03 | 1.5982E-03 | 4.9504E-04 | 4.1072E 02 | 4.5071E-01 |

ELEMENT COVARIANCE MATRIX 1

| A | E | I | O | U | T |
|---|---|---|---|---|---|
| 1.9875E 06 | | | | | |
| -5.0636E-02 | 1.4594E-09 | | | | |
| 3.6845E-01 | -5.6941E-09 | 2.0411E-07 | | | |
| -1.6046E 00 | 1.0965E-07 | 1.4652E-06 | 3.0715E-05 | | |
| -5.6203E 02 | 1.2874E-05 | -1.2667E-04 | -1.0367E-04 | 1.7372E-01 | |
| -8.4275E 03 | 1.9407E-04 | -1.8751E-03 | -1.1217E-03 | 2.5961E 00 | 3.8801E 01 |
| SQUARE ROOT OF DIAGONAL | | | | | |
| 1.4098E 03 | 3.8201E-05 | 4.5178E-04 | 5.5421E-03 | 4.1680E-01 | 6.2290E 00 |

EXAMPLE 15 (continued)

```
PERIOD, APOGEE, PERIGEE, COVARIANCE MATRIX 1
   PERIOD        APOGEE        PERIGEE
  2.6968E-01
  3.3524E 02   4.9501E 05
  1.1290E 03   1.3252E 06   4.8047E 06
SQUARE ROOT OF DIAGONAL
  5.1930E-01   7.0357E 02   2.1920E 03


CARTESIAN COVARIANCE MATRIX 1
      X             Y             Z            XDOT          YDOT          ZDOT
  2.1562E 05
  1.3939E 04   4.9471E 04
  1.2054E 05  -5.7608E 04   2.1268E 05
  4.5393E 02   2.2719E 01   2.7516E 02   9.6513E-01
  7.9805E 01   1.4352E 02  -1.3754E 02   1.4966E-01   4.4062E-01
  9.1184E 02  -8.5410E 01   8.1958E 02   1.9876E 00  -8.0566E-02   4.6203E 00
SQUARE ROOT OF DIAGONAL
  4.6435E 02   2.2242E 02   4.6117E 02   9.8241E-01   6.6380E-01   2.1495E 00


ORBIT PLANE COVARIANCE MATRIX 2
     XI            ETA          ZETA          XIDOT         ETADOT       ZETADOT
  2.1564E 07
  4.3203E 07   8.7405E 07
 -1.3652E 08  -2.7566E 08   8.7042E 08

  3.2810E 04   6.5627E 04  -2.0736E 05   4.9960E 01
  7.0178E 04   1.4162E 05  -4.4677E 05   1.0668E 02   2.2966E 02
  1.1869E 05   2.4155E 05  -7.6020E 05   1.8015E 02   3.9085E 02   6.7123E 02
SQUARE ROOT OF DIAGONAL
  4.6437E 03   9.3490E 03   2.9503E 04   7.0682E 00   1.5154E 01   2.5908E 01


    VOLUME 1           VOLUME 2
    0.35935468E 10     0.10869521E 02
```

## EXAMPLE 15 (continued)

```
POLAR COVARIANCE MATRIX 2
     ALPHA         DELTA          BETA           A              R              V
  4.0623E-05
 -5.1870E-04   6.7387E-03
 -8.0952E-05   1.0600E-03   1.6893E-04
 -1.8581E-05   2.3719E-04   3.6560E-05   8.6257E-06
  2.9144E 01  -3.7985E 02  -6.0289E 01  -1.3202E 01   2.1564E 07
  9.5762E-02  -1.2400E 00  -1.9525E-01  -4.3655E-02   6.9996E 04   2.2848E 02
SQUARE ROOT OF DIAGONAL
  6.3736E-03   8.2090E-02   1.2997E-02   2.9370E-03   4.6437E 03   1.5115E 01


ELEMENT COVARIANCE MATRIX  2
      A            E             I             O             U             T
SQUARE ROOT OF DIAGONAL
  3.4603E 04   6.1300E-04   4.1249E-02   2.5811E-01   1.2483E 01   1.9049E 02


PERIOD, APOGEE, PERIGEE, COVARIANCE MATRIX 2
    PERIOD        APOGEE        PERIGEE
  1.6246E 02
  2.7564E 05   4.6779E 08
  6.0647E 05   1.0288E 09   2.2640E 09
SQUARE ROOT OF DIAGONAL
  1.2746E 01   2.1629E 04   4.7582E 04


CARTESIAN COVARIANCE MATRIX 2
       X             Y             Z            XDOT          YDOT          ZDOT
  3.8742E 07
  3.8602E 07   3.8699E 07
  1.8651E 08   1.8663E 08   9.0195E 08

 -3.9590E 04  -4.0084E 04  -1.9345E 05   4.2586E 01
  1.2462E 05   1.2488E 05   6.0238E 05  -1.2929E 02   4.0303E 02
 -1.3796E 05  -1.3918E 05  -6.7106E 05   1.4633E 02  -4.4901E 02   5.0523E 02
SQUARE ROOT OF DIAGONAL
  6.2243E 03   6.2208E 03   3.0032E 04   6.5258E 00   2.0076E 01   2.2477E 01
```

EXAMPLE 15 (continued)

```
PARTIALS  P WRT Q
    0.21955343E 01     -0.12087670E 02     0.13324642E-00     0.48450407E 01     0.94387812E 04

    0.73602600E 02     -0.12963595E 02    -0.12826756E 02

PARTIALS  P WRT Q
    0.17523052E-05      0.43918582E-04    -0.18651530E-04    -0.16287667E-04    -0.20039122E 01

   -0.76948032E-02      0.46895918E-04     0.47333626E-04
```

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# APPENDIX

## Standard Values of Constants and Parameters

Included in this appendix are lists of the contents of the arrays OBJZ, OBJT, OBLT, INTEG, C, NUMB, and IFLAG. The first three contain the earth gravitational model used; INTEG contains the numerical integration parameters; C contains various constants. The non-zero values given for these arrays comprise a 'standard' set of values to be input. The two arrays NUMB and IFLAG contain many items that are equivalent to input items described in Section 5. There are also additional items that the user may want to alter.

```
EARTH MODEL

OBJZ - ZONAL COEFFICIENTS                              EQUIV (OBJZ,C(8))
  1      N1          4.
  2     J(2)      1082.3 E-6
  3     J(3)        -2.3 E-6
  4     J(4)        -1.8 E-6
  5     J(5)         0.  E-6

OBJT - TESSERAL COEFFICIENTS                           EQUIV (OBJT,PINES)
  1      N2=2.      N2=3.      N2=4.
  2     J(2,1)     J(2,1)     J(2,1)
  3     J(2,2)     J(3,1)     J(3,1)
  4                J(2,2)     J(4,1)
  5                J(3,2)     J(2,2)
  6                J(3,3)     J(3,2)
  7                           J(4,2)
  8                           J(3,3)
  9                           J(4,3)
 10                           J(4,4)

OBLT - TESSERAL LONGITUDES                             EQUIV (OBLT,DRAG(11),OPRAM(2))
       THE LONGITUDES ARE INPUT IN THE SAME LOCATIONS RELATIVE TO OBLT AS THE
       CORRESPONDING COEFFICIENTS RELATIVE TO OBJT.

NOTE - IN THE STANDARD USE OF TRACE ONLY THE ZONAL COEFFICIENTS
       ARE ASSIGNED NON-ZERO VALUES - SEE ABOVE.
```

Figure A-1. Earth Model

```
INTEG   PARAMETERS OF THE TRAJECTORY INTEGRATION, WITH RECOMMENDED VALUES

 1       1      FORMULATION. 1 - COWELL (EQS. OF MOTION).
 2       2      DIFFERENTIAL EQUATION SUBROUTINE.  1 - AMRK, 2 - COW
 3              (NOT USEO)
 4       1      SUN          (4-9) ARE SELECTORS FOR OTHER-BODY PERTURBATIONS
 5       1      MOON         IF TAPE UNIT NUMBER AT NUMB(18) IS NOT ZERO. THEN
 6       0      VENUS        PERTURBATIONS ARE INCLUOEO OR OMITTED ACCORDING
 7       0      MARS         AS THE SELECTOR IS NON-ZERO OR ZERO.
 8       0      JUPITER
 9       0      SATURN
10     C(22)    SUN          (10-15) ARE INTERPOLATION SCALE FACTORS FOR USE
11 1.00002516   MOON         WITH EPHEMERIS TAPES.  THEY ARE SET IN CSET
12     C(22)    VENUS        USING CONTENTS OF C(22), THE NUMBER OF EARTH-
13     C(22)    MARS         RADII IN AN ASTRONOMICAL UNIT.
14     C(22)    JUPITER
15     C(22)    SATURN
16
THRU            (NOT USED)
22
23     1E-10    E BAR, COW SUBROUTINE TRUNCATION ERROR CONTROL PARAMETER.
24     5E-8     E BAR (X)   AMRK POSITION TRUNCATION ERROR  PARAMETER.
25     2E-9     E BAR (OX)  AMRK VELOCITY TRUNCATION ERROR  PARAMETER.
26       1.     A, SEE COW WRITEUP.
30       1.     INITIAL TIME STEP SIZE.
31       0      MINIMUM TIME STEP SIZE.
32       4.     MAXIMUM TIME STEP SIZE.
33     1E-7     KEPLER EQUATION CONVERGENCE CRITERION.
34       4      RATIO OF COWELL STEP SIZE TO RUNGE-KUTTA STEP SIZE.
35       1      IF 1, OO NOT RECOMPUTE PERTURBATIONS FOR CORRECTOR. IF 2, DO.
36              FLAG FOR PERTURBATION COMPUTATION  (USED IF INTEG(35)=1).
37     .001     LEAST SQUARES CONVERGENCE CRITERION (RELATIVE)
38     .001     LEAST SQUARES CONVERGENCE CRITERION (ABSOLUTE)
```

Figure A-2. Numerical Integration Parameters

```
C     CONSTANTS.

 1 .0043752691     EARTH ROTATION RATE (RAD/MIN)
 2 .0055303935     GM, EARTH GRAVITATION CONSTANT (ER**3/MIN**2)
 3                 DBSERVATION TIME CDRRECTIDN FACTOR
 4                 B=1-E, RELATIVE SEMI-MINOR AXIS DF ELLIPSOID (COMP. IN CSET)
 5                 B**2/A**2 = (1-E)**2  (COMPUTED IN CSET)
 6      .5         FACTDR FOR DECREASING BOUNDS IN L.S. SOLUTIDN
 7                 2*E-E**2  (COMPUTED IN CSET)
 8-12              (RESERVED FOR EARTH MODEL COEFFICIENTS - SEE LISTING)
13                 INDICATOR FOR DATA EDITING
14  57.2957795     ANGLE CONVERSION FACTOR
15   20925738.     A, EARTH RADIUS IN FEET
16    332951.3     RELATIVE MASS OF SUN
17   .0122999        MOON
18   .814979         VENUS
19   .107821         MARS
20   317.887         JUPITER
21   95.129          SATURN
22   23454.865     EARTH-RADII PER ASTRONOMICAL UNIT
23    3443.9336    NAUTICAL MILES (6076.1155 FT) PER EARTH-RADIUS
24   20925738.     I/D DISTANCE CONVERSION FACTOR
25   348762.30     I/O VELOCITY CONVERSION FACTDR
26                 (NOT USED)
27                 (RESERVED FOR DELTA-T CONVERSION FACTDR)
28                 ANGLE FDR L1,L2,L3 - RADIANS  (COMPUTED IN CSET FRDM C(35))
29                 CONSTANT FOR DOPPLER RATE
30   348762.30     FT/SEC PER ER/MIN
31     1.5         FACTOR FOR INCREASING BOUNDS IN L.S. SOLUTION
32   1.0471976     RAD/MIN PER DEG/SEC
33   3.14159265    PI
34     298.3       RECIPROCAL OF E=ELLIPTICITY
35                 ANGLE FOR L1,L2,L3 IN DEGREES.
36   82505.922     DEG/DAY PER RAD/MIN
37-39              DIRECTIDN COSINES OF BODY AXIS FOR LOOK ANGLE
40                 APPROX TIME STEP FOR RISE - SET PREDICTION (=2 IF NOT INPUT)
41                 NO. OF REVS PREDICTED FOR. (=1 IF NOT INPUT)
49                 ANALYTIC TRAJECTORY INDICATOR
50-100             CONSTANTS AND TABULAR DATA FDR ATMDSPHERE MDDEL
```

Figure A-3. Constants

| NUMB | NUMBER OF - | SET IN |
|---|---|---|
| 1 | RADAR STATIONS | TRAIN |
| 2 | OBSERVATION TIMES | TRAIN |
| | OR STATIONS REQUIRING SIGHTING EPHEMERIS DATA | DUM |
| | OR STATIONS REQUIRING SIMULATION DATA | DUM |
| 3 | WORDS IN COMPACTED RADAR OBSERVATION LIST - TOTAL | TRAIN |
| 4 | WORDS OF COMPACTED RADAR OBSERVATIONS IN CORE NOW | TRAIN |
| 5 | DIFFERENTIAL EQUATION PARAMETERS TO BE SOLVED FOR | CHAIN |
| 6 | INITIAL CONDITION PARAMETERS TO BE SOLVED FOR | CHAIN |
| 7 | RADAR STATION PARAMETERS TO BE SOLVED FOR | KING |
| 8 | RADAR OBSERVATION PARAMETERS TO BE SOLVED FOR | KING |
| 9 | PROGRAM TAPE UNIT | REIN |
| 10 | MAXIMUM ITERATIONS ALLOWED | CHAIN |
| 11 | TOTAL PARAMETERS (SUM OF 5,6,7 AND 8) | TRAIN |
| 12 | TRAJECTORY PARAMETERS (SUM OF 5 AND 6) | TRAIN |
| 13 | OBSERVATIONS (TOTAL NUMBER OF MEASUREMENTS) | TRAIN |
| 14 | PRESENT ITERATION | MAIN |
| 15 | BASIC TYPES OF OBSERVATIONS | CHAIN |
| 16 | RADAR STATIONS IN SOS LIST - 9 OR LESS (STATIONS FOR WHICH INDIVIDUAL SOS IS OBTAINED) | TRAIN |
| 17 | TOTAL RADAR PARAMETERS (SUM OF 7 AND 8) | TRAIN |
| 18 | TAPE UNIT FOR PLANETARY COORDINATE TAPE | CHAIN |
| 19 | SECOND ORDER DIFFERENTIAL EQUATIONS BEING INTEGRATED (3*(1+NUMB(12)*IFLAG(8))) | INCN |
| 20 | POSSIBLE KINDS OF RADAR PARAMETERS (LAT, LONG, HEIGHT, AND BIASES) | CHAIN |
| 21 | WORDS IN CORE FOR SIGHTING EPHEMERIS BUCKET | CHAIN |
| 22 | POSSIBLE KINDS OF SIGHTING DATA | CHAIN |
| 23 | DATA NOISE CONTROL (ZERO FOR NO NOISE, NON-ZERO STARTS RANDOM NUMBER GENERATOR FOR DATA NOISE) | CHAIN |
| 24 | POSITION IN ITIN LIST OF FUNCTION BEING EXECUTED | ITIN |
| 25 | EFFECTIVE PARAMETERS BEING SOLVED FOR (NUMB(11)-(NO. OF CONSTRAINTS)) | CHAIN |
| 26 | TAPE UNIT FOR GENERATING BCD RADAR STATION AND OBSERVATION TAPE (IF ZERO, NO TAPE GENERATED) | CHAIN |
| 27 | FLOCKS OF DATA | TRAIN |
| 28 | TAPE UNIT FOR ECLIPSE COORD TAPE | CHAIN |
| 29 | CURRENT PASS THRU DUM - 0 FOR 1ST PASS, 1 FOR REST. | CHAIN |
| 30 | ELEMENTS IN ATA | MAIN |

Figure A-4. Numerical Parameters

```
IFLAG - OPTION INDICATORS

 1   CURRENT FUNCTION BEING EXECUTED (1-TRAIN, 2-TRACKING,
       3-TRAJECTORY,4-GAIN, 5-FEIGN)
 2   RESTORE (1) LAST GOOO SOLUTION OR CORRECT (0) PRESENT SOLUTION
 3   SOLVE NORMAL EQUATIONS (0) OR OO NOT SOLVE (NON-ZERO)
 4   REASON FOR EXIT FROM MAIN (1-MAXIMUM NUMBER OF ITERATIONS,
       2-CONVERGED, 3-TRAJECTORY COMPLETEO)
 5   CORRECTIONS ARE HITTING BOUNDS (1) OR NOT (0)
 6   COMPLETE SIGHTING EPHEMERIS (0) OR RISE AND SET TIMES ONLY (NON-ZERO)
 7       OPTION FOR GAIN AND FEIGN INPUT
                           -1  READ ALL STAT LOC ANO OATA
         IF IFLAG(7) = 0  NO REAO - ALL SAME AS LAST CASE
                            1  READ OATA ONLY - STAT LOC SAME AS LAST CASE
         NOT USEO IF NUMB(29)=0
 8   ANALYTIC TRAJECTORY PARTIALS (0) OR VARIATIONAL EQUATIONS (1)
 9   BOUNOS PROVIOEO FOR LEAST SQUARES SOLUTION (1) OR NO BOUNOS (0)
10   NORMAL MATRIX COMPUTED IN LEGS (-1) OR ACCUMULATED IN LAYR (1)
11   T-MATRIX OPTION  IF=0, NO T-MATRIX
                       =1, INPUT ORHODH*H/RHO,  NO EARTH FLATTENING
                       =2, INPUT DRHOOH*H/RHO, USE EARTH FLATTENING
                       =3, CALC. DRHODH*H/RHO,  NO EARTH FLATTENING
                       =4, CALC. DRHODH*H/RHO, USE EARTH FLATTENING
12   ITIN - PARAMETER SPECIFYING SEQUENCE OF FUNCTIONS TO BE
       PERFORMED.
14   CONSOLE CONTROL - SOME ITEMS PRINTED ON-LINE IF NON-ZERO
15   IF ITIN = 5  THEN -1 = HOLD ATA FROM PREVIOUS CASE.
                        0 = NO ATA OR INVERSE INPUT.
                        1 = INPUT ATA AT ATA(1).
                        2 = INPUT INVERSE AT ATA(501).
     IF ITIN = 3  THEN  0 = NORMAL
                       +X = X IS THE LOGICAL TAPE UNIT ON WHICH THE
                            TRAJECTORY IS WRITTEN.
                       -X = THE CURRENT TRAJECTORY IS DIFFERENCED WITH THE
                            ONE ON TAPE UNIT X AND THE OIFFERENCES ARE
                            PLACED ON TAPE UNIT X+1.
```

Figure A-5. Option Indicators

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*(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)*

| | |
|---|---|
| 1 ORIGINATING ACTIVITY (Corporate author)<br>Aerospace Corporation<br>El Segundo, California | 2a REPORT SECURITY CLASSIFICATION<br>Unclassified<br>2b GROUP |

3 REPORT TITLE

TRACE - Aerospace Orbit Determination Program

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

5 AUTHOR(S) (Last name, first name, initial)

Mercer, R. J., et al.

| 6. REPORT DATE | 7a. TOTAL NO. OF PAGES | 7b. NO. OF REFS |
|---|---|---|
| November 1964 | 234 | 18 |

| | |
|---|---|
| 8a. CONTRACT OR GRANT NO.<br>AF 04(695)-269<br>b. PROJECT NO.<br>c.<br>d. | 9a. ORIGINATOR'S REPORT NUMBER(S)<br>TDR-269(4110-04)-1<br>9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)<br>SSD-TDR-64-159 |

10 AVAILABILITY/LIMITATION NOTICES

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| 11. SUPPLEMENTARY NOTES | 12. SPONSORING MILITARY ACTIVITY<br>Space Systems Division<br>Air Force Systems Command<br>Los Angeles, California |

13 ABSTRACT

TRACE is a multiple-purpose satellite orbit-determination program for the IBM 7090 computer. Its applications include: (1) prediction - the generation of a satellite trajectory and associated ground trace and station sighting data; (2) orbit determination - estimation of trajectory parameters, station locations, and observational biases, so as to best fit a set of observations; and (3) error analysis - estimation of the potential accuracy attainable by a tracking system, given the station locations, the data types, rates and quality, the uncertainties in the model parameters, and the specifications of the nominal orbit. The report contains the objectives of the program, some theoretical foundations, the equations and methods employed, the structure of the program, and complete instructions for its use.

| 14. KEY WORDS |
| --- |
| Orbit Determination<br>Differential Correction<br>TRACE<br>Tracking Program<br>Least Squares<br>Variational Equations<br>Tracking System Analysis |

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